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Interactions of a single massless tensor field with the mixed symmetry (3,1). No-go results

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Abstract: Non-trivial consistent interactions of a single massless tensor field $t_{\lambda\mu\nu|\alpha}$ with the mixed symmetry corresponding to the two-column Young diagram (3,1), dual to linearized gravity in $D = 6$ space-time dimensions, are studied in the following cases: self-couplings, cross-interactions with a Pauli-Fierz field and cross-couplings to purely matter theories. Our approach is based on the deformation of the solution to the master equation from the antifield-BRST formalism by means of the local cohomology of the BRST differential. Our main results, obtained under the assumptions that the emerging interactions are local, smooth, Lorentz-covariant, Poincaré-invariant and of maximum derivative order equal to two, can be synthesized into: no consistent self-couplings exist; no cross-interactions with the Pauli-Fierz field can be added; no non-trivial consistent cross-couplings to the matter theories are allowed.

Keywords: BRST Symmetry, BRST Quantization.

Contents

1. Introduction

Tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [1]–[5], appear in the context of many physically interesting theories, like superstrings, supergravities or supersymmetric high spin theories. Such models held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher [6]–[11], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [12] or a lagrangian first-order approach [13, 14] to some classes of massless or partially massive mixed symmetry-type tensor gauge fields, suggestively resembling to the tetrad formalism of General Relativity. An important matter related to mixed symmetry-type tensor fields is the study of their consistent interactions, among themselves, as well as with higher-spin gauge theories [15]–[18]. Beyond any doubt, the best approach to this problem is the cohomological one, based on the deformation of the solution to the master equation [19]. The purpose of this paper is to investigate the smooth, local, consistent interactions that involve a single free massless tensor gauge field $t_{\lambda\mu\nu|\alpha}$ with the mixed symmetry of a two-column (3,1) Young diagram in three distinct situations: self-couplings, interactions with the massless spin-two field, described in the free limit by the Pauli-Fierz action [20], and couplings to purely matter theories. It is worth mentioning the duality of this model to the Pauli-Fierz theory in $D = 6$ dimensions and, in this respect, the recent developments concerning the dual formulations of linearized gravity from the perspective of M-theory [21, 22, 23]. Although the appropriate algebraic-differential settings for handling gauge theories involving exotic representations of the Lorentz group are those of hyperforms [24] and multiforms, the latter one being related to the N-complexes associated with higher spin gauge theories [25], we will avoid their usage in extenso here due to the rather few indices carried by the relevant tensor fields.

Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques. For each situation, we initially determine the associated free antifield-BRST symmetry s, which splits as the sum between the Koszul-Tate differential and the exterior longitudinal derivative only, $s = \delta + \gamma$. Then, we solve the basic equations of the deformation procedure with the help of the corresponding local BRST cohomology. Under the supplementary assumptions of Lorentz covariance, Poincaré invariance and the maximum derivative order of the interacting lagrangians being equal to two, we prove the following no-go results:

- there are no consistent and non-trivial self-interactions for the tensor $t_{\lambda\mu\nu|\alpha}$;

- there are no consistent cross-interactions between such a tensor field and the Pauli-Fierz model. Only the Pauli-Fierz theory gives rise to consistent self-interactions, described by the Einstein-Hilbert action with a cosmological term, invariant under diffeomorphisms;

- there are no cross-couplings to purely matter fields.

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The paper is organized in eight sections. In section 2 we expose the lagrangian formulation of the free massless tensor gauge field $t_{\lambda\mu\nu|\alpha}$ and in section 3 we focus on the construction of the associated BRST differential. Section 4 briefly reviews the main ingredients of the antifield-BRST deformation procedure, while the following three sections are devoted to the study of the consistent deformations for this type of mixed symmetry tensor field in the context of self-interactions, cross-couplings to the Pauli-Fierz field, and respectively cross-interactions with purely matter theories. Section 8 ends the paper with the main conclusions. The paper also contains an appendix section, dedicated to the proofs of some useful cohomological results related to the tensor field $t_{\lambda\mu\nu|\alpha}$.

2. Free model: lagrangian action and gauge invariance

We begin with the lagrangian action

$$S_0\left[t_{\lambda\mu\nu|\alpha}\right] = \int d^Dx \left(\frac{1}{2}\left(\left(\partial^\rho t^{\lambda\mu\nu|\alpha}\right)\left(\partial_\rho t_{\lambda\mu\nu|\alpha}\right) - \left(\partial_\alpha t^{\lambda\mu\nu|\alpha}\right)\left(\partial^\beta t_{\lambda\mu\nu|\beta}\right)\right) - \right.$$
$$- \frac{3}{2}\left(\left(\partial_\lambda t^{\lambda\mu\nu|\alpha}\right)\left(\partial^\rho t_{\rho\mu\nu|\alpha}\right) + \left(\partial^\rho t^{\lambda\mu}\right)\left(\partial_\rho t_{\lambda\mu}\right)\right) +$$
$$\left. + 3\left(\left(\partial_\alpha t^{\lambda\mu\nu|\alpha}\right)\left(\partial_\lambda t_{\mu\nu}\right) + \left(\partial_\rho t^{\rho\mu}\right)\left(\partial^\lambda t_{\lambda\mu}\right)\right)\right), \tag{2.1}$$

in $D \geq 5$ space-time dimensions, where $t_{\lambda\mu\nu|\alpha}$ is a tensor field with the mixed symmetry of the two-column Young diagram

$$t_{\lambda\mu\nu|\alpha} = \begin{array}{ll} \lambda & \alpha \\ \mu & \\ \nu & \end{array} . \tag{2.2}$$

It is thus completely antisymmetric in its first three indices

$$t_{\lambda\mu\nu|\alpha} = -t_{\mu\lambda\nu|\alpha} = -t_{\lambda\nu\mu|\alpha} = -t_{\nu\mu\lambda|\alpha}\,, \tag{2.3}$$

and satisfies the identity

$$t_{[\lambda\mu\nu|\alpha]} \equiv 0\,. \tag{2.4}$$

Here and in the sequel the notation $[\lambda\mu\cdots\nu]$ signifies antisymmetrization with respect to all the indices between brackets, but without normalization factors. For instance, the left-hand side of (2.4) is equal by our convention to

$$t_{[\lambda\mu\nu|\alpha]} = t_{\lambda\mu\nu|\alpha} - t_{\lambda\mu\alpha|\nu} + t_{\nu\alpha\lambda|\mu} - t_{\nu\alpha\mu|\lambda}\,. \tag{2.5}$$

The trace of $t_{\lambda\mu\nu|\alpha}$ is defined by

$$t_{\lambda\mu} = \sigma^{\nu\alpha} t_{\lambda\mu\nu|\alpha}\,, \tag{2.6}$$

and it is obviously an antisymmetric tensor

$$t_{\lambda\mu} = -t_{\mu\lambda}\,. \tag{2.7}$$

Everywhere in this paper we employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (-,++++\cdots)$.

A generating set of gauge transformations for the action (2.1) is given by

$$\begin{aligned} \delta_{\epsilon,\chi} t_{\lambda\mu\nu|\alpha} &= 3\partial_\alpha \epsilon_{\lambda\mu\nu} + \partial_{[\lambda}\, \epsilon_{\,\mu\nu]\alpha} + \partial_{[\lambda}\, \chi_{\,\mu\nu]|\alpha} \\ &= -3\partial_{[\lambda}\, \epsilon_{\,\mu\nu\alpha]} + 4\partial_{[\lambda}\, \epsilon_{\,\mu\nu]\alpha} + \partial_{[\lambda}\, \chi_{\,\mu\nu]|\alpha}\,, \end{aligned} \tag{2.8}$$

where the gauge parameters $\epsilon_{\lambda\mu\nu}$ determine a completely antisymmetric tensor

$$\epsilon_{\lambda\mu\nu} = \begin{array}{c} \lambda \\ \mu \\ \nu \end{array}, \tag{2.9}$$

and the other set of gauge parameters, $\chi_{\mu\nu|\alpha}$, displays the mixed symmetry associated with the two-column Young diagram

$$\chi_{\mu\nu|\alpha} = \begin{array}{cc} \mu & \alpha \\ \nu & \end{array}, \tag{2.10}$$

such that they are antisymmetric in the first two indices

$$\chi_{\mu\nu|\alpha} = -\chi_{\nu\mu|\alpha}\,, \tag{2.11}$$

and satisfy the identity

$$\chi_{[\mu\nu|\alpha]} \equiv 0\,. \tag{2.12}$$

(Coming back to our antisymmetrization convention, there are only four terms in the first piece from the right-hand side of (2.8) (the second line), $\partial_{[\lambda}\, \epsilon_{\,\mu\nu\alpha]} = \partial_\lambda \epsilon_{\mu\nu\alpha} - \partial_\mu \epsilon_{\lambda\nu\alpha} + \partial_\nu \epsilon_{\alpha\lambda\mu} - \partial_\alpha \epsilon_{\nu\lambda\mu}$, and, along the same line, the left-hand side of (2.12) contains three terms $\chi_{[\mu\nu|\alpha]} = \chi_{\mu\nu|\alpha} + \chi_{\nu\alpha|\mu} + \chi_{\alpha\mu|\nu}$.) It is easy to see that the identity (2.12) is necessary in order to ensure that the gauge variation of the tensor field also obeys the identity (2.4) checked by the field itself, $\delta_{\epsilon,\chi} t_{[\lambda\mu\nu|\alpha]} \equiv 0$. We observe that if in (2.8) we make the transformations

$$\epsilon_{\mu\nu\alpha} \to \epsilon^{(\omega,\psi)}_{\mu\nu\alpha} = -\frac{1}{2}\partial_{[\mu}\, \omega_{\,\nu\alpha]}, \tag{2.13}$$

$$\chi_{\mu\nu|\alpha} \to \chi^{(\omega,\psi)}_{\mu\nu|\alpha} = \partial_{[\mu}\, \psi_{\,\nu]\alpha} + 2\partial_\alpha \omega_{\mu\nu} - \partial_{[\mu}\, \omega_{\,\nu]\alpha}\,, \tag{2.14}$$

with $\omega_{\nu\alpha}$ antisymmetric and $\psi_{\nu\alpha}$ symmetric (but otherwise arbitrary)

$$\omega_{\nu\alpha} = -\omega_{\alpha\nu}\,, \qquad \psi_{\nu\alpha} = \psi_{\alpha\nu}\,, \tag{2.15}$$

then the gauge variation of the tensor field identically vanishes

$$\delta_{\epsilon^{(\omega,\psi)},\chi^{(\omega,\psi)}} t_{\lambda\mu\nu|\alpha} \equiv 0\,. \tag{2.16}$$

Moreover, if in (2.13)–(2.14) we perform the changes

$$\omega_{\nu\alpha} \to \omega^{(\theta)}_{\nu\alpha} = \partial_{[\mu}\, \theta_{\,\nu]}, \tag{2.17}$$

$$\psi_{\nu\alpha} \to \psi^{(\theta)}_{\nu\alpha} = -3\partial_{(\mu}\, \theta_{\,\nu)}, \tag{2.18}$$

with θ_ν an arbitrary vector field, where $(\mu\nu\cdots)$ signifies symmetrization with respect to the indices between parentheses without normalization factors, $\partial_{(\mu}\theta_{\nu)} = \partial_\mu\theta_\nu + \partial_\nu\theta_\mu$, then the transformed gauge parameters (2.13)–(2.14) identically vanish

$$\epsilon_{\mu\nu\alpha}^{\left(\omega^{(\theta)},\psi^{(\theta)}\right)} \equiv 0\,, \qquad \chi_{\mu\nu|\alpha}^{\left(\omega^{(\theta)},\psi^{(\theta)}\right)} \equiv 0\,. \tag{2.19}$$

Meanwhile, there is no non-vanishing local transformation of θ_ν that simultaneously annihilates $\omega_{\nu\alpha}^{(\theta)}$ and $\psi_{\nu\alpha}^{(\theta)}$ of the form (2.17)–(2.18), and hence no further local reducibility identity. All these allow us to conclude that the generating set of gauge transformations (2.8) is off-shell second-stage reducible. It is obvious that the accompanying gauge algebra is abelian.

The field equations associated with action (2.1) are

$$\Sigma : \frac{\delta S_0}{\delta t_{\lambda\mu\nu|\alpha}} \equiv -T^{\lambda\mu\nu|\alpha} \approx 0\,, \tag{2.20}$$

where (minus) the Euler-Lagrange derivatives have the form

$$\begin{aligned} T^{\lambda\mu\nu|\alpha} &= \Box t^{\lambda\mu\nu|\alpha} - \partial_\rho\left(\partial^{[\lambda}t^{\mu\nu]\rho|\alpha} + \partial^\alpha t^{\lambda\mu\nu|\rho}\right) + \partial^\alpha\partial^{[\lambda}t^{\mu\nu]} + \\ &\quad + \sigma^{\alpha[\lambda}\left(\partial_\rho\left(\partial_\beta t^{\mu\nu]\rho|\beta} - \partial^\mu t^{\nu]\rho}\right) - \Box t^{\mu\nu]}\right). \end{aligned} \tag{2.21}$$

(We stress that, according to our convention, the notation $\partial^{[\lambda}t^{\mu\nu]\rho|\alpha}$ involves three terms, $\sigma^{\alpha[\lambda}\partial_\rho\partial^\mu t^{\nu]\rho}$ comprises six terms, etc.) The tensor $T^{\lambda\mu\nu|\alpha}$ has the same properties like the field $t^{\lambda\mu\nu|\alpha}$, being antisymmetric in its first three indices and satisfying the identity

$$T^{[\lambda\mu\nu|\alpha]} \equiv 0\,. \tag{2.22}$$

It is also useful to compute its trace

$$T^{\lambda\mu} \equiv \sigma_{\nu\alpha}T^{\lambda\mu\nu|\alpha} = (4-D)\left(\Box t^{\lambda\mu} + \partial_\rho\left(\partial^{[\lambda}t^{\mu]\rho} - \partial_\alpha t^{\lambda\mu\rho|\alpha}\right)\right), \tag{2.23}$$

which is an antisymmetric tensor

$$T^{\lambda\mu} = -T^{\mu\lambda}\,. \tag{2.24}$$

The gauge invariance of the action (2.1) under the transformations (2.8) shows that not all the Euler-Lagrange derivatives (2.21) are independent, but rather satisfy the Noether identities

$$4\partial_\alpha T^{\lambda\mu\nu|\alpha} = 0,\ \partial_\lambda\left(3T^{\lambda\mu\nu|\alpha} - T^{\mu\nu\alpha|\lambda}\right) = 0\,, \tag{2.25}$$

while the reducibility of the generating set of gauge transformations indicates that not all the Noether identities (2.25) are independent. We will alternatively use the equivalent simpler form

$$\partial_\alpha T^{\lambda\mu\nu|\alpha} = 0\,, \qquad \partial_\lambda T^{\lambda\mu\nu|\alpha} = 0 \tag{2.26}$$

of (2.25), which we will also call the Noether identities of the action (2.1). We note that (2.26) induces a corresponding identity at the level of the trace of $T^{\lambda\mu\nu|\alpha}$, namely

$$\partial_\lambda T^{\lambda\mu} = 0\,. \tag{2.27}$$

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The number of physical degrees of freedom for the theory under consideration is

$$N_{\text{phys}} = \frac{D\,(D-2)\,(D-3)\,(D-5)}{8}\,, \tag{2.28}$$

so $D = 6$ is the critical dimension for the theory to have local degrees of freedom. This is also the dimension for which action (2.1) is dual to the Pauli-Fierz theory. The structure of the field equations together with the behavior of the generating set of gauge transformations enable us to state that the free mixed symmetry-type tensor field under discussion is described by a linear gauge theory of Cauchy order equal to four.

The most general quantity invariant under the gauge transformations (2.8) is second-order in the derivatives of the tensor field

$$F^{\lambda\mu\nu\xi|\alpha\beta} = \partial^\alpha \partial^{[\lambda} t^{\mu\nu\xi]|\beta} - \partial^\beta \partial^{[\lambda} t^{\mu\nu\xi]|\alpha}, \tag{2.29}$$

and accordingly we will call it the curvature tensor. It exhibits the mixed symmetry of the two-column Young diagram

$$F^{\lambda\mu\nu\xi|\alpha\beta} = \begin{array}{cc} \lambda & \alpha \\ \mu & \beta \\ \nu & \\ \xi & \end{array}\,, \tag{2.30}$$

such that it is separately antisymmetric in its first four indices and respectively in the last two ones, and fulfills the (algebraic) Bianchi I identity

$$F^{[\lambda\mu\nu\xi|\alpha]\beta} \equiv 0\,. \tag{2.31}$$

Meanwhile, the curvature tensor obeys two types of (differential) Bianchi II identities

$$\partial^{[\kappa} F^{\lambda\mu\nu\xi]|\alpha\beta} \equiv 0\,, \qquad F^{\lambda\mu\nu\xi|[\alpha\beta,\gamma]} \equiv 0\,, \tag{2.32}$$

where we made the notation

$$F^{\lambda\mu\nu\xi|\alpha\beta,\gamma} \equiv \partial^\gamma F^{\lambda\mu\nu\xi|\alpha\beta}\,. \tag{2.33}$$

Its traces are defined through

$$\begin{aligned} F^{\lambda\mu\nu|\alpha} &\equiv \sigma_{\xi\beta} F^{\lambda\mu\nu\xi|\alpha\beta} = \Box t^{\lambda\mu\nu|\alpha} - \partial_\rho \partial^{[\lambda} t^{\mu\nu]\rho|\alpha} - \\ &\quad - \partial^\alpha \left(\partial_\beta t^{\lambda\mu\nu|\beta} - \partial^{[\lambda} t^{\mu\nu]} \right), \end{aligned} \tag{2.34}$$

$$F^{\lambda\mu} \equiv \sigma_{\nu\alpha} F^{\lambda\mu\nu|\alpha} = 2\left(\Box t^{\lambda\mu} + \partial_\rho \left(\partial^{[\lambda} t^{\mu]\rho} - \partial_\alpha t^{\lambda\mu\rho|\alpha} \right) \right), \tag{2.35}$$

with $F^{\lambda\mu\nu|\alpha}$ having the same mixed symmetry like the original tensor field and $F^{\lambda\mu}$ being antisymmetric

$$F^{\lambda\mu\nu|\alpha} = -F^{\mu\lambda\nu|\alpha} = -F^{\lambda\nu\mu|\alpha} = -F^{\nu\mu\lambda|\alpha}\,, \qquad F^{[\lambda\mu\nu|\alpha]} \equiv 0\,, \qquad F^{\lambda\mu} = -F^{\mu\lambda}\,. \tag{2.36}$$

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There exists a strong relationship between the curvature tensor (2.29) and the theory (2.1) subject to the gauge invariance (2.8), in the sense that on the one hand the functions (2.21) that define the field equations (2.20) can be expressed only in terms of (2.34)–(2.35) and on the other hand the Noether identities (2.26) are consequences of the Bianchi II identities (2.32). Indeed, we have that

$$T^{\lambda\mu\nu|\alpha} = F^{\lambda\mu\nu|\alpha} - \frac{1}{2}\sigma^{\alpha[\lambda}F^{\mu\nu]}\,, \tag{2.37}$$

and therefore the field equations (2.20) are completely equivalent with the vanishing of the simple trace (2.34)

$$T^{\lambda\mu\nu|\alpha} \approx 0 \quad \Leftrightarrow \quad F^{\lambda'\mu'\nu'|\alpha} \approx 0\,. \tag{2.38}$$

In the meantime, if we apply ∂_α and respectively ∂_λ on (2.37) and perform the necessary summations, we get that

$$\partial_\alpha T^{\lambda\mu\nu|\alpha} = \partial_\alpha F^{\lambda\mu\nu|\alpha} - \frac{1}{2}\partial^{[\lambda}F^{\mu\nu]}\,, \tag{2.39}$$

$$\partial_\lambda T^{\lambda\mu\nu|\alpha} = \partial_\lambda F^{\lambda\mu\nu|\alpha} - \frac{1}{2}\partial^\alpha F^{\mu\nu} - \frac{1}{2}\sigma^{\alpha[\mu}\partial_\lambda F^{\nu]\lambda}\,. \tag{2.40}$$

Next, we start from the former Bianchi II identity in (2.32) on which we apply $\frac{1}{2}\sigma_{\kappa\alpha}\sigma_{\xi\beta}$

$$\partial^{[\kappa}F^{\lambda\mu\nu\xi]|\alpha\beta} \equiv 0 \quad \Rightarrow \quad \frac{1}{2}\sigma_{\kappa\alpha}\sigma_{\xi\beta}\partial^{[\kappa}F^{\lambda\mu\nu\xi]|\alpha\beta} = \partial_\alpha F^{\lambda\mu\nu|\alpha} - \frac{1}{2}\partial^{[\lambda}F^{\mu\nu]} \equiv 0\,, \tag{2.41}$$

then act with $\frac{1}{2}\sigma_{\lambda\gamma}\sigma_{\xi\beta}$ on the latter one

$$F^{\lambda\mu\nu\xi|[\alpha\beta,\gamma]} \equiv 0 \quad \Rightarrow \quad \frac{1}{2}\sigma_{\lambda\gamma}\sigma_{\xi\beta}F^{\lambda\mu\nu\xi|[\alpha\beta,\gamma]} = \partial_\lambda F^{\lambda\mu\nu|\alpha} - \frac{1}{2}\partial^\alpha F^{\mu\nu} \equiv 0\,, \tag{2.42}$$

and finally with $\frac{1}{3}\sigma_{\lambda\gamma}\sigma_{\xi\beta}\sigma_{\mu\alpha}$ also on the latter identity

$$F^{\lambda\mu\nu\xi|[\alpha\beta,\gamma]} \equiv 0 \quad \Rightarrow \quad \frac{1}{3}\sigma_{\lambda\gamma}\sigma_{\xi\beta}\sigma_{\mu\alpha}F^{\lambda\mu\nu\xi|[\alpha\beta,\gamma]} = \partial_\lambda F^{\nu\lambda} \equiv 0\,. \tag{2.43}$$

Now, we observe that (2.41)–(2.43) express the identically vanishing of the right-hand sides of the relations (2.39)–(2.40) due to the Bianchi II identities (2.32). Accordingly, the left-hand sides of (2.39)–(2.40) will also be strongly equal to zero, providing thus nothing but the Noether identities (2.26) of the action (2.1).

The free model (2.1) can be consistently interpreted in terms of an operator $\bar{d}$ that is third-order nilpotent [7, 25], $\bar{d}^3 = 0$, and acts on the vector space $T_2(\mathcal{M})$ of tensor fields with the mixed symmetry corresponding to two-column Young diagrams, defined on a pseudo-riemannian manifold $\mathcal{M}$ of dimension D. For instance, both the curvature tensor $F_{\lambda\mu\nu\xi|\alpha\beta}$ and the original tensor field $t_{\lambda\mu\nu|\alpha}$ belong to this vector space and are correlated one to the other through the operator $\bar{d}$ via the proportionality relation

$$F_{\lambda\mu\nu\xi|\alpha\beta} \sim \left(\bar{d}^2 t\right)_{\lambda\mu\nu\xi\alpha\beta}\,. \tag{2.44}$$

Moreover, the Bianchi II identities (2.32) follow from the third-order nilpotency of $\bar{d}$, $\bar{d}F \sim \bar{d}^3 t \equiv 0$. Along the same line, we observe that the formula (2.37) relates the functions defining the free field equations (2.20) to the curvature tensor by a generalized Hodge-duality. The generalized cohomology of $\bar{d}$ on $T_2(\mathcal{M})$ when $\mathcal{M}$ has the trivial topology of $\mathbb{R}^D$ together with this type of generalized Hodge-duality reveal many important features of the free model under study. For example, if $\bar{T}^{\lambda\mu\nu|\alpha}$ is a covariant tensor field with mixed symmetry of the two-column Young diagram (2.2) and satisfies the equations

$$\partial_\lambda \bar{T}^{\lambda\mu\nu|\alpha} = 0,\ \partial_\alpha \bar{T}^{\lambda\mu\nu|\alpha} = 0\,, \tag{2.45}$$

then there exists a tensor $\bar{\Phi}^{\rho\lambda\mu\nu|\beta\alpha}$ with the mixed symmetry of the curvature (corresponding to the Young diagram in (2.30)), such that

$$\bar{T}^{\lambda\mu\nu|\alpha} = \partial_\xi \partial_\beta \bar{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}\,. \tag{2.46}$$

It is easy to check the above statement in connection with the functions (2.21) that define the field equations for the model under consideration. Indeed, direct computation provides

$$T^{\lambda\mu\nu|\alpha} = \partial_\xi \partial_\beta \Phi^{\lambda\mu\nu\xi|\alpha\beta}\,, \tag{2.47}$$

where

$$\begin{aligned}\Phi^{\lambda\mu\nu\xi|\alpha\beta} = & -\sigma^{\alpha[\lambda}\sigma^{\mu]\beta}t^{\nu\xi} - \sigma^{\alpha[\nu}\sigma^{\xi]\beta}t^{\lambda\mu} + \sigma^{\alpha[\lambda}\sigma^{\nu]\beta}t^{\mu\xi} + \\ & + \sigma^{\alpha[\mu}\sigma^{\xi]\beta}t^{\lambda\nu} - \sigma^{\alpha[\lambda}\sigma^{\xi]\beta}t^{\mu\nu} - \sigma^{\alpha[\mu}\sigma^{\nu]\beta}t^{\lambda\xi} + \\ & + \sigma^{\alpha[\lambda}t^{\mu\nu\xi]|\beta} - \sigma^{\beta[\lambda}t^{\mu\nu\xi]|\alpha}\,,\end{aligned} \tag{2.48}$$

and the corresponding $\Phi^{\lambda\mu\nu\xi|\alpha\beta}$ obviously satisfies the required mixed symmetry. Each of the last two quantities from the right-hand side of (2.48) comprises four terms.

3. Antibracket-BRST symmetry of the free model

The construction of the antifield-BRST symmetry for this free theory debuts with the identification of the algebra on which the BRST differential s acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields. The ghost spectrum for the model under study comprises the fermionic ghosts $\left\{\eta_{\lambda\mu\nu}, \mathcal{G}_{\mu\nu|\alpha}\right\}$ associated with the gauge parameters $\left\{\epsilon_{\lambda\mu\nu}, \chi_{\mu\nu|\alpha}\right\}$ from (2.8), the bosonic ghosts for ghosts $\{C_{\mu\nu}, \mathcal{C}_{\nu\alpha}\}$ due to the first-stage reducibility parameters $\{\omega_{\mu\nu}, \psi_{\nu\alpha}\}$ in (2.13)–(2.14), as well as the fermionic ghost for ghost for ghost C_ν corresponding to the second-stage reducibility parameter θ_ν in (2.17)–(2.18). In order to make compatible the behavior of the gauge and reducibility parameters with that of the accompanying ghosts, we ask that $\eta_{\lambda\mu\nu}$ and $C_{\mu\nu}$ are completely antisymmetric, $\mathcal{G}_{\mu\nu|\alpha}$ obeys the analogue of the properties (2.11)–(2.12), and $\mathcal{C}_{\nu\alpha}$ is symmetric

$$\eta_{\lambda\mu\nu} = -\eta_{\mu\lambda\nu} = -\eta_{\lambda\nu\mu} = -\eta_{\nu\mu\lambda}\,, \qquad \mathcal{G}_{\mu\nu|\alpha} = -\mathcal{G}_{\nu\mu|\alpha}\,, \qquad \mathcal{G}_{[\mu\nu|\alpha]} \equiv 0\,, \tag{3.1}$$

$$C_{\mu\nu} = -C_{\nu\mu}\,, \qquad \mathcal{C}_{\nu\alpha} = \mathcal{C}_{\alpha\nu}\,. \tag{3.2}$$

The antifield spectrum is organized into the antifield $t^{*\lambda\mu\nu|\alpha}$ of the original tensor field, together with those of the ghosts, $\left\{\eta^{*\lambda\mu\nu}, \mathcal{G}^{*\mu\nu|\alpha}\right\}$, $\{C^{*\mu\nu}, \mathcal{C}^{*\nu\alpha}\}$ and respectively $C^{*\nu}$, of statistics opposite to that of the associated field/ghost. It is understood that $t^{*\lambda\mu\nu|\alpha}$ is subject to some conditions similar to (2.3)–(2.4)

$$t^{*\lambda\mu\nu|\alpha} = -t^{*\mu\lambda\nu|\alpha} = -t^{*\lambda\nu\mu|\alpha} = -t^{*\nu\mu\lambda|\alpha},\ t^{*[\lambda\mu\nu|\alpha]} = 0\,, \tag{3.3}$$

and, along the same line, it is required that

$$\eta^{*\lambda\mu\nu} = -\eta^{*\mu\lambda\nu} = -\eta^{*\lambda\nu\mu} = -\eta^{*\nu\mu\lambda}\,, \qquad \mathcal{G}^{*\mu\nu|\alpha} = -\mathcal{G}^{*\nu\mu|\alpha}\,, \qquad \mathcal{G}^{*[\mu\nu|\alpha]} \equiv 0\,, \tag{3.4}$$

$$C^{*\mu\nu} = -C^{*\nu\mu}\,, \qquad \mathcal{C}^{*\nu\alpha} = \mathcal{C}^{*\alpha\nu}\,. \tag{3.5}$$

For subsequent purpose, we denote the trace of $t^{*\lambda\mu\nu|\alpha}$ by $t^{*\lambda\mu}$, being understood that it is antisymmetric

$$t^{*\lambda\mu} = \sigma_{\nu\alpha} t^{*\lambda\mu\nu|\alpha}\,, \qquad t^{*\lambda\mu} = -t^{*\mu\lambda}\,. \tag{3.6}$$

Since both the gauge generators and reducibility functions for this model are field-independent, it follows that the BRST differential s simply reduces to

$$s = \delta + \gamma\,, \tag{3.7}$$

where δ represents the Koszul-Tate differential, graded by the antighost number agh (agh$\,(\delta) = -1$), and γ stands for the exterior derivative along the gauge orbits, whose degree is named pure ghost number pgh (pgh$\,(\gamma) = 1$). Naturally, these two degrees do not interfere (agh$\,(\gamma) = 0$, pgh$\,(\delta) = 0$). The overall degree that grades the BRST complex is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that gh$\,(s) =$ gh$\,(\delta) =$ gh$\,(\gamma) = 1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued like

$$\mathrm{pgh}\left(t_{\lambda\mu\nu|\alpha}\right) = 0\,, \qquad \mathrm{pgh}\left(\eta_{\lambda\mu\nu}\right) = 1 = \mathrm{pgh}\left(\mathcal{G}_{\mu\nu|\alpha}\right), \tag{3.8}$$

$$\mathrm{pgh}\left(C_{\mu\nu}\right) = 2 = \mathrm{pgh}\left(\mathcal{C}_{\nu\alpha}\right), \qquad \mathrm{pgh}\left(C_{\nu}\right) = 3\,, \tag{3.9}$$

$$\mathrm{pgh}\left(t^{*\lambda\mu\nu|\alpha}\right) = \mathrm{pgh}\left(\eta^{*\lambda\mu\nu}\right) = \mathrm{pgh}\left(\mathcal{G}^{*\mu\nu|\alpha}\right) = 0\,, \tag{3.10}$$

$$\mathrm{pgh}\left(C^{*\mu\nu}\right) = \mathrm{pgh}\left(\mathcal{C}^{*\nu\alpha}\right) = \mathrm{pgh}\left(C^{*\nu}\right) = 0\,, \tag{3.11}$$

$$\mathrm{agh}\left(t_{\lambda\mu\nu|\alpha}\right) = \mathrm{agh}\left(\eta_{\lambda\mu\nu}\right) = \mathrm{agh}\left(\mathcal{G}_{\mu\nu|\alpha}\right) = 0\,, \tag{3.12}$$

$$\mathrm{agh}\left(C_{\mu\nu}\right) = \mathrm{agh}\left(\mathcal{C}_{\nu\alpha}\right) = \mathrm{agh}\left(C_{\nu}\right) = 0\,, \tag{3.13}$$

$$\mathrm{agh}\left(t^{*\lambda\mu\nu|\alpha}\right) = 1\,, \qquad \mathrm{agh}\left(\eta^{*\lambda\mu\nu}\right) = 2 = \mathrm{agh}\left(\mathcal{G}^{*\mu\nu|\alpha}\right), \tag{3.14}$$

$$\mathrm{agh}\left(C^{*\mu\nu}\right) = 3 = \mathrm{agh}\left(\mathcal{C}^{*\nu\alpha}\right), \qquad \mathrm{agh}\left(C^{*\nu}\right) = 4. \tag{3.15}$$

Actually, (3.7) is a decomposition of the BRST differential according to the antighost number and it shows that s contains only components of antighost number equal to minus one and zero. Consequently, the equation expressing the nilpotency of s projected on the distinct values of the antighost number is equivalent with the nilpotency and anticommutation of its components

$$s^2 = 0 \quad \Leftrightarrow \quad \left(\delta^2 = 0\,,\ \delta\gamma + \gamma\delta = 0\,,\ \gamma^2 = 0\right). \tag{3.16}$$

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The Koszul-Tate differential is imposed to realize an homological resolution of the algebra of smooth functions defined on the stationary surface of field equations (2.20), while the exterior longitudinal derivative is related to the gauge symmetries (2.8) of the action (2.1) through its cohomology at pure ghost number zero computed in the cohomology of δ, which is required to be the algebra of physical observables for the free model under consideration. The actions of δ and γ on the generators from the BRST complex, which enforce all the above mentioned properties, are given by

$$\begin{aligned}
\gamma t_{\lambda\mu\nu|\alpha} &= 3\partial_\alpha \eta_{\lambda\mu\nu} + \partial_{[\lambda}\eta_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}_{\mu\nu]|\alpha} \\
&= -3\partial_{[\lambda}\eta_{\mu\nu\alpha]} + 4\partial_{[\lambda}\eta_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}_{\mu\nu]|\alpha}\,,
\end{aligned} \tag{3.17}$$

$$\gamma\eta_{\lambda\mu\nu} = -\frac{1}{2}\partial_{[\lambda}C_{\mu\nu]}\,, \tag{3.18}$$

$$\begin{aligned}
\gamma\mathcal{G}_{\mu\nu|\alpha} &= 2\partial_\alpha C_{\mu\nu} - \partial_{[\mu}C_{\nu]\alpha} + \partial_{[\mu}\mathcal{C}_{\nu]\alpha} \\
&= 2\partial_{[\mu}C_{\nu\alpha]} - 3\partial_{[\mu}C_{\nu]\alpha} + \partial_{[\mu}\mathcal{C}_{\nu]\alpha}\,,
\end{aligned} \tag{3.19}$$

$$\gamma C_{\mu\nu} = \partial_{[\mu}C_{\nu]}\,, \qquad \gamma\mathcal{C}_{\nu\alpha} = -3\partial_{(\nu}C_{\alpha)}\,, \qquad \gamma C_\nu = 0\,, \tag{3.20}$$

$$\gamma t^{*\lambda\mu\nu|\alpha} = \gamma\eta^{*\lambda\mu\nu} = \gamma\mathcal{G}^{*\mu\nu|\alpha} = \gamma C^{*\mu\nu} = \gamma\mathcal{C}^{*\nu\alpha} = \gamma C^{*\nu} = 0\,, \tag{3.21}$$

$$\delta t_{\lambda\mu\nu|\alpha} = \delta\eta_{\lambda\mu\nu} = \delta\mathcal{G}_{\mu\nu|\alpha} = \delta C_{\mu\nu} = \delta\mathcal{C}_{\nu\alpha} = \delta C_\nu = 0\,, \tag{3.22}$$

$$\delta t^{*\lambda\mu\nu|\alpha} = T^{\lambda\mu\nu|\alpha}\,, \qquad \delta\eta^{*\lambda\mu\nu} = -4\partial_\alpha t^{*\lambda\mu\nu|\alpha}, \tag{3.23}$$

$$\delta\mathcal{G}^{*\mu\nu|\alpha} = -\partial_\lambda\left(3t^{*\lambda\mu\nu|\alpha} - t^{*\mu\nu\alpha|\lambda}\right), \tag{3.24}$$

$$\delta C^{*\mu\nu} = 3\partial_\lambda\left(\mathcal{G}^{*\mu\nu|\lambda} - \frac{1}{2}\eta^{*\lambda\mu\nu}\right), \qquad \delta\mathcal{C}^{*\nu\alpha} = \partial_\mu\mathcal{G}^{*\mu(\nu|\alpha)}\,, \tag{3.25}$$

$$\delta C^{*\nu} = 6\partial_\mu\left(\mathcal{C}^{*\mu\nu} - \frac{1}{3}C^{*\mu\nu}\right), \tag{3.26}$$

where $T^{\lambda\mu\nu|\alpha}$ is given in (2.21). By convention, we take δ and γ to act like right derivations. We note that the action of the Koszul-Tate differential on the antifields with the antighost numbers equal to two and three gain a simpler expression if we perform the changes of variables

$$\mathcal{G}'^{*\mu\nu|\alpha} = \mathcal{G}^{*\mu\nu|\alpha} + \frac{1}{4}\eta^{*\mu\nu\alpha}\,, \qquad \mathcal{C}'^{*\nu\alpha} = \mathcal{C}^{*\nu\alpha} - \frac{1}{3}C^{*\nu\alpha}\,, \tag{3.27}$$

$$\mathrm{agh}\left(\mathcal{G}'^{*\mu\nu|\alpha}\right) = 2\,, \qquad \mathrm{agh}\left(\mathcal{C}'^{*\nu\alpha}\right) = 3\,, \tag{3.28}$$

where $\mathcal{G}'^{*\mu\nu|\alpha}$ is still antisymmetric in its first two indices, but does not fulfill any longer the last identity in (3.4), and $\mathcal{C}'^{*\nu\alpha}$ has no definite symmetry or antisymmetry properties. With the help of (3.23)–(3.26), we find that δ acts on the transformed antifields through the relations

$$\delta\mathcal{G}'^{*\mu\nu|\alpha} = -3\partial_\lambda t^{*\lambda\mu\nu|\alpha}\,, \qquad \delta\mathcal{C}'^{*\nu\alpha} = 2\partial_\mu\mathcal{G}'^{*\mu\nu|\alpha}\,, \qquad \delta C^{*\nu} = 6\partial_\mu\mathcal{C}'^{*\mu\nu}\,. \tag{3.29}$$

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The same observation is valid for γ if we make the changes

$$\mathcal{G}'_{\mu\nu|\alpha} = \mathcal{G}_{\mu\nu|\alpha} + 4\eta_{\mu\nu\alpha}\,, \qquad \mathcal{C}'_{\nu\alpha} = \mathcal{C}_{\nu\alpha} - 3C_{\nu\alpha}\,, \tag{3.30}$$

$$\mathrm{pgh}\left(\mathcal{G}'_{\mu\nu|\alpha}\right) = 1\,, \qquad \mathrm{pgh}\left(\mathcal{C}'_{\nu\alpha}\right) = 2, \tag{3.31}$$

in terms of which we can write

$$\gamma t_{\lambda\mu\nu|\alpha} = -\frac{1}{4}\partial_{[\lambda}\mathcal{G}'_{\mu\nu|\alpha]} + \partial_{[\lambda}\mathcal{G}'_{\mu\nu]|\alpha}\,, \qquad \gamma\mathcal{G}'_{\mu\nu|\alpha} = \partial_{[\mu}\mathcal{C}'_{\nu]\alpha}\,, \qquad \gamma\mathcal{C}'_{\nu\alpha} = -6\partial_\nu C_\alpha\,. \tag{3.32}$$

Again, $\mathcal{G}'_{\mu\nu|\alpha}$ is antisymmetric in its first two indices and does not satisfy the last identity in (3.1), while $\mathcal{C}'_{\nu\alpha}$ has no definite symmetry or antisymmetry.[1]

The lagrangian BRST differential admits a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s\cdot = (\cdot, S)$, where $(\;,\;)$ signifies the antibracket and S denotes the canonical generator of the BRST symmetry. It is a bosonic functional of ghost number zero, involving both field/ghost and antifield spectra, that obeys the master equation

$$(S,S) = 0\,. \tag{3.33}$$

The master equation is equivalent with the second-order nilpotency of s, where its solution S encodes the entire gauge structure of the associated theory. Taking into account the formulas (3.17)–(3.26), as well as the standard actions of δ and γ in canonical form, we find that the complete solution to the master equation for the model under study is given by

$$\begin{aligned} S = S_0\left[t_{\lambda\mu\nu|\alpha}\right] + \int d^Dx \Big(& t^{*\lambda\mu\nu|\alpha}\left(3\partial_\alpha\eta_{\lambda\mu\nu} + \partial_{[\lambda}\eta_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}_{\mu\nu]|\alpha}\right) - \\ & - \frac{1}{2}\eta^{*\lambda\mu\nu}\partial_{[\lambda}C_{\mu\nu]} + \mathcal{G}^{*\mu\nu|\alpha}\left(2\partial_\alpha C_{\mu\nu} - \partial_{[\mu}C_{\nu]\alpha} + \partial_{[\mu}\mathcal{C}_{\nu]\alpha}\right) + \\ & + C^{*\mu\nu}\partial_{[\mu}C_{\nu]} - 3\mathcal{C}^{*\nu\alpha}\partial_{(\nu}C_{\alpha)}\Big), \end{aligned} \tag{3.34}$$

such that it contains pieces with the antighost number ranging from zero to three.

The main ingredients of the antifield-BRST symmetry derived in this section will be useful in the sequel at the analysis of consistent interactions that can be added to the action (2.1) without changing its number of independent gauge symmetries.

4. Brief review of the antifield-BRST deformation procedure

There are three main types of consistent interactions that can be added to a given gauge theory: (i) the first type deforms only the lagrangian action, but not its gauge transformations, (ii) the second kind modifies both the action and its transformations, but not the gauge algebra, and (iii) the third, and certainly most interesting category, changes everything, namely, the action, its gauge symmetries and the accompanying algebra.

[1]We have deliberately chosen the same notations for the transformed variables (3.27) and (3.30). They actually form pairs that are conjugated in the antibracket, $\left(\mathcal{G}'_{\mu\nu|\alpha}, \mathcal{G}'^{*\mu_1\nu_1|\alpha_1}\right) = \frac{1}{2}\delta^{[\mu_1}_{\mu}\delta^{\nu_1]}_{\nu}\delta^{\alpha_1}_{\alpha}$, $\left(\mathcal{C}'_{\nu\alpha}, \mathcal{C}'^{*\nu_1\alpha_1}\right) = \delta^{\nu_1}_{\nu}\delta^{\alpha_1}_{\alpha}$ (in De Witt condensed notations).

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The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into the solution of the master equation for the interacting theory

$$\bar{S} = S + gS_1 + g^2 S_2 + O\left(g^3\right) , \qquad \varepsilon\left(\bar{S}\right) = 0 , \qquad \mathrm{gh}\left(\bar{S}\right) = 0 , \tag{4.1}$$

such that

$$\left(\bar{S}, \bar{S}\right) = 0 . \tag{4.2}$$

Here and in the sequel $\varepsilon(F)$ denotes the Grassmann parity of F. The projection of (4.1) on the various powers of the coupling constant induces the following tower of equations:

$$g^0 : \ (S, S) = 0 , \tag{4.3}$$

$$g^1 : \ (S_1, S) = 0 , \tag{4.4}$$

$$g^2 : \ \frac{1}{2}(S_1, S_1) + (S_2, S) = 0 , \tag{4.5}$$

$$\vdots$$

The first equation is satisfied by hypothesis. The second one governs the first-order deformation of the solution to the master equation, S_1, and it expresses the fact that S_1 is a BRST co-cycle, $sS_1 = 0$, and hence it exists and is local. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as space-time locality, are imposed. Obviously, only non-trivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to trivial deformations of the initial theory, and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that S_1 is a non-trivial BRST-observable, $S_1 \in H^0(s)$ ($H^0(s)$ denotes the cohomology space of the BRST differential at ghost number zero). Once that the deformation equations (4.4)–(4.5), etc., have been solved by means of specific cohomological techniques, from the consistent non-trivial deformed solution to the master equation we can extract all the information on the gauge structure of the resulting interacting theory.

5. Self-interactions

The first aim of our paper is to investigate the consistent interactions that can be added to the action (2.1) without modifying either the field/ghost/antifield spectrum or the number of independent gauge symmetries. This matter is addressed in the context of the antifield-BRST deformation procedure described in the above and relies on computing the solutions to the equations (4.4)–(4.5), etc., from the cohomology of the BRST differential. For obvious reasons, we consider only smooth, local and manifestly covariant deformations, and, meanwhile, restrict to Poincaré-invariant quantities, i.e., we do not allow explicit dependence of the space-time co-ordinates. If we make the notation $S_1 = \int d^D x \, a$, with a a local function, then the local form of the equation (4.4), which we have seen that controls the first-order deformation of the solution to the master equation, becomes

$$sa = \partial_\mu m^\mu \,, \qquad \mathrm{gh}\,(a) = 0 \,, \qquad \varepsilon\,(a) = 0 \,, \tag{5.1}$$

for some local m^μ, and it shows that the non-integrated density of the first-order deformation pertains to the local cohomology of s at ghost number zero, $a \in H^0\,(s|d)$, where d denotes the exterior space-time differential. In order to analyze the above equation, we develop a according to the antighost number

$$a = \sum_{k=0}^{I} a_k \,, \qquad \mathrm{agh}\,(a_k) = k \,, \qquad \mathrm{gh}\,(a_k) = 0 \,, \qquad \varepsilon\,(a_k) = 0 \,, \tag{5.2}$$

and assume, without loss of generality, that the above decomposition stops at some finite value of the antighost number, I. (This can be shown, for instance, like in [26] (section 3), under the sole assumption that the interacting lagrangian at the first order in the coupling constant, a_0, has a finite, but otherwise arbitrary, derivative order.) By taking into account the splitting (3.7) of the BRST differential, the equation (5.1) is equivalent to a tower of local equations, corresponding to the different decreasing values of the antighost number

$$\gamma a_I = \partial_\mu \stackrel{(I)^{\mu}}{m} \,, \tag{5.3}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \stackrel{(I-1)^{\mu}}{m} \,, \tag{5.4}$$

$$\delta a_k + \gamma a_{k-1} = \partial_\mu \stackrel{(k-1)^{\mu}}{m} \,, \qquad I-1 \geq k \geq 1 \,, \tag{5.5}$$

where $\left(\stackrel{(k)^{\mu}}{m}\right)_{k=\overline{0,I}}$ are some local currents with $\mathrm{agh}\left(\stackrel{(k)^{\mu}}{m}\right) = k$. It can be proved[2] that we can replace the equation (5.3) at strictly positive antighost numbers with

$$\gamma a_I = 0 \,, \qquad \mathrm{agh}\,(a_I) = I > 0 \,. \tag{5.6}$$

In conclusion, under the assumption that $I > 0$, the representative of highest antighost number from the non-integrated density of the first-order deformation can always be taken to be γ-closed, such that the equation (5.1) associated with the local form of the first-order deformation is completely equivalent to the tower of equations (5.6) and (5.4)–(5.5).

Before proceeding to the analysis of the solutions to the first-order deformation equations, we briefly comment on the uniqueness and triviality of such solutions. Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to the top equation (5.6) is clearly unique up to γ-exact contributions,

$$a_I \rightarrow a_I + \gamma b_I \,, \qquad \mathrm{agh}\,(b_I) = I \,, \qquad \mathrm{pgh}\,(b_I) = I-1 \,, \qquad \varepsilon\,(b_I) = 1 \,. \tag{5.7}$$

Meanwhile, if it turns out that a_I reduces to γ-exact terms only, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. In other words, the non-triviality of the first-order deformation a is translated at its highest antighost number component into the requirement that

$$a_I \in H^I\,(\gamma) \,, \tag{5.8}$$

[2]The proof will be given after introducing the notion of "invariant polynomial", in the appendix A, corollary 1.

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where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ at pure ghost number equal to I. At the same time, the general condition on the non-integrated density of the first-order deformation to be in a non-trivial cohomological class of $H^0(s|d)$ shows on the one hand that the solution to (5.1) is unique up to s-exact pieces plus divergences

$$a \to a + sb + \partial_\mu n^\mu\,, \qquad \mathrm{gh}(b) = -1\,, \qquad \varepsilon(b) = 1\,, \qquad \mathrm{gh}(n^\mu) = 0\,, \qquad \varepsilon(n^\mu) = 0\,, \tag{5.9}$$

and on the other hand that if the general solution to (5.1) is found to be completely trivial, $a = sb + \partial_\mu n^\mu$, then it can be made to vanish, $a = 0$.

In the light of the above discussion, we now investigate the solutions to the equations (5.6) and (5.4)–(5.5). We have seen that the solution to (5.6) belongs to the cohomology of the exterior longitudinal derivative (see the formula (5.8)), such that we need to compute $H(\gamma)$ in order to construct the component of highest antighost number from the first-order deformation. This matter is solved with the help of the definitions (3.17)–(3.21).

5.1 $H(\gamma)$ and $H(\delta|d)$

From (3.17) we observe that γ acts on the original tensor field through a gauge transformation (2.8) with the gauge parameters replaced by the ghosts, such that the γ-closed quantities constructed out of the tensor gauge field $t_{\lambda\mu\nu|\alpha}$ and its space-time derivatives are nothing but the gauge-invariant objects of the theory (2.1). As it was discussed in section 2, the only such objects are the curvature tensor (2.29) $F_{\lambda\mu\nu\xi|\alpha\beta}$ and its derivatives, and thus they all belong to $H(\gamma)$. The definitions (3.21) allows us to identify all the antifields

$$\Pi^{*\Delta} \equiv \left(t^{*\lambda\mu\nu|\alpha}, \mathcal{G}'^{*\mu\nu|\alpha}, \mathcal{C}'^{*\nu\alpha}, C^{*\nu}\right), \tag{5.10}$$

and their derivatives as being also γ-closed elements, such that these pertain to $H(\gamma)$.[3] (We choose to work with the prime antifields $\mathcal{G}'^{*\mu\nu|\alpha}$ and $\mathcal{C}'^{*\nu\alpha}$ since they cover all the independent components of the Koszul-Tate generators $\left(\eta^{*\lambda\mu\nu}, \mathcal{G}^{*\mu\nu|\alpha}\right)$ and respectively $\left(C^{*\mu\nu}, \mathcal{C}'^{*\nu\alpha}\right)$ at antighost number two and three.)

Related to the pure ghost number one ghosts $\left\{\eta_{\lambda\mu\nu}, \mathcal{G}_{\mu\nu|\alpha}\right\}$, we use the definitions (3.18)–(3.19) and observe that there is no γ-closed linear combination of these undifferentiated ghosts belonging to $H(\gamma)$. On the other hand, it is easy to see that the only γ -closed linear objects involving just their first-order derivatives are

$$\mathcal{F}_{\lambda\mu\nu\alpha} \equiv \partial_{[\lambda}\eta_{\mu\nu\alpha]}\,, \qquad \mathcal{T}_{\lambda\mu\nu|\alpha} \equiv 4\partial_{[\lambda}\eta_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}_{\mu\nu]|\alpha}\,. \tag{5.11}$$

Clearly, $\mathcal{F}_{\lambda\mu\nu\alpha}$ is completely antisymmetric, while $\mathcal{T}_{\lambda\mu\nu|\alpha}$ is antisymmetric in its first three indices and does not satisfy the identity $\mathcal{T}_{[\lambda\mu\nu|\alpha]} \equiv 0$. Then, $\mathcal{T}_{\lambda\mu\nu|\alpha}$ can be decomposed in

[3] We notice that neither the curvature nor the antifields nor any of their derivatives are trivial in $H(\gamma)$ since they all have the pure ghost number equal to zero and there is no object in the BRST algebra with strictly negative values of this degree.

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terms of two tensors belonging to irreducible representations of $GL(D,\mathbb{R})$, namely

$$\mathcal{T}_{\lambda\mu\nu|\alpha} = \begin{array}{c}\lambda\\ \mu\\ \nu\end{array} \otimes \; \alpha \; = \begin{array}{c}\lambda\\ \mu\\ \nu\\ \alpha\end{array} + \begin{array}{cc}\lambda & \alpha\\ \mu & \\ \nu & \end{array} \equiv \mathcal{T}'_{\lambda\mu\nu\alpha} + \mathcal{T}''_{\lambda\mu\nu|\alpha}\,, \tag{5.12}$$

where $\mathcal{T}'_{\lambda\mu\nu\alpha}$ is completely antisymmetric in its indices and $\mathcal{T}''_{\lambda\mu\nu|\alpha}$ is antisymmetric in its first three indices and satisfies the identity $\mathcal{T}''_{[\lambda\mu\nu|\alpha]} \equiv 0$. It can be checked that $\mathcal{T}'_{\lambda\mu\nu\alpha}$ and $\mathcal{T}''_{\lambda\mu\nu|\alpha}$ read as

$$\mathcal{T}'_{\lambda\mu\nu\alpha} = \frac{1}{4}\mathcal{T}_{[\lambda\mu\nu|\alpha]}\,, \qquad \mathcal{T}''_{\lambda\mu\nu|\alpha} = \mathcal{T}_{\lambda\mu\nu|\alpha} - \frac{1}{4}\mathcal{T}_{[\lambda\mu\nu|\alpha]}\,, \tag{5.13}$$

where, according to our antisymmetrization convention, $\mathcal{T}_{[\lambda\mu\nu|\alpha]}$ reduces to four terms only. Due to the properties (3.1), $\mathcal{T}'_{\lambda\mu\nu\alpha}$ is proportional with $\mathcal{F}_{\lambda\mu\nu\alpha}$

$$\mathcal{T}'_{\lambda\mu\nu\alpha} = 3\mathcal{F}_{\lambda\mu\nu\alpha}\,, \tag{5.14}$$

such that we can write

$$\mathcal{T}_{\lambda\mu\nu|\alpha} = 3\mathcal{F}_{\lambda\mu\nu\alpha} + \mathcal{T}''_{\lambda\mu\nu|\alpha}\,. \tag{5.15}$$

On the one hand, the definition (3.17) can be rewritten in terms of $\mathcal{T}'$, $\mathcal{T}''$ and $\mathcal{F}$ like

$$\gamma t_{\lambda\mu\nu|\alpha} = \mathcal{T}''_{\lambda\mu\nu|\alpha}\,. \tag{5.16}$$

On the other hand, $\mathcal{F}_{\lambda\mu\nu\alpha}$ is not γ -exact

$$\mathcal{F}_{\lambda\mu\nu\alpha} \neq \gamma f_{\lambda\mu\nu\alpha}(t)\,. \tag{5.17}$$

(This can be shown directly assuming that $\mathcal{F}_{\lambda\mu\nu\alpha}$ can be written like a γ-exact combination of undifferentiated tensor field components with real coefficients, which yields to an incompatible algebraic linear system for the coefficients.) As a consequence of (5.15)–(5.17), we can state that the independent and γ-non-exact quantities in (5.11) reduce to the completely antisymmetric tensor $\mathcal{F}_{\lambda\mu\nu\alpha}$, and thus its components span all the non-trivial objects of pure ghost one from $H(\gamma)$ involving only the first-order derivatives of the ghosts $\{\eta_{\lambda\mu\nu}, \mathcal{G}_{\mu\nu|\alpha}\}$. By direct computation, we find that the first-order derivatives of $\mathcal{F}_{\lambda\mu\nu\alpha}$ (and, of course, those of $\mathcal{T}_{\lambda\mu\nu|\alpha}$) are γ-exact

$$\partial_\rho \mathcal{F}_{\lambda\mu\nu\alpha} = \gamma\left(\frac{1}{3}\partial_{[\lambda}t_{\mu\nu\alpha]|\rho}\right), \tag{5.18}$$

$$\partial_\rho \mathcal{T}_{\lambda\mu\nu|\alpha} = \gamma\left(\partial_\rho t_{\lambda\mu\nu|\alpha} - \partial_\alpha t_{\lambda\mu\nu|\rho} + \partial_{[\lambda}t_{\mu\nu]\alpha|\rho}\right), \tag{5.19}$$

and so will be all the higher-order ones. In conclusion, the only non-trivial linear combinations in $H(\gamma)$ constructed out the ghosts of pure ghost number equal to one $\{\eta_{\lambda\mu\nu}, \mathcal{G}_{\mu\nu|\alpha}\}$ and their derivatives are given by $\mathcal{F}_{\lambda\mu\nu\alpha}$, defined in the former relation from (5.11). The result that $H(\gamma)$ contains non-trivial elements containing antisymmetrized derivatives of the ghosts is not singular among the models with mixed symmetry-type tensor fields. The same happens in the case of the Pauli-Fierz theory (see section 6) or of the tensor $t_{\lambda\mu|\alpha}$ [12].

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Next, we approach $H(\gamma)$ in the context of the pure ghost number two ghosts $\{C_{\mu\nu}, \mathcal{C}_{\nu\alpha}\}$ with the help of the first two definitions in (3.20). First, we notice that there is no linear combination of these undifferentiated ghosts in $H(\gamma)$. Second, we observe that the only linear, γ -closed combinations built with their derivatives are

$$\mathcal{F}_{\lambda\mu\nu} \equiv \partial_{[\lambda} C_{\mu\nu]}\,, \qquad \mathcal{R}_{\mu\nu|\alpha\beta} \equiv \partial_\mu\partial_\alpha\mathcal{C}_{\nu\beta} - \partial_\nu\partial_\alpha\mathcal{C}_{\mu\beta} - \partial_\mu\partial_\beta\mathcal{C}_{\nu\alpha} + \partial_\nu\partial_\beta\mathcal{C}_{\mu\alpha}\,, \tag{5.20}$$

where $\mathcal{F}_{\lambda\mu\nu}$ is completely antisymmetric and $\mathcal{R}_{\mu\nu|\alpha\beta}$ displays the same mixed symmetry like the Riemann tensor from General Relativity

$$\mathcal{R}_{\mu\nu|\alpha\beta} = -\mathcal{R}_{\nu\mu|\alpha\beta} = -\mathcal{R}_{\mu\nu|\beta\alpha} = \mathcal{R}_{\alpha\beta|\mu\nu}\,, \qquad \mathcal{R}_{[\mu\nu|\alpha]\beta} \equiv 0\,. \tag{5.21}$$

However, the relations (3.18)–(3.19) yield both $\mathcal{F}_{\lambda\mu\nu}$ and $\mathcal{R}_{\mu\nu|\alpha\beta}$ as being γ-trivial

$$\mathcal{F}_{\lambda\mu\nu} = \gamma\left(-2\eta_{\lambda\mu\nu}\right), \tag{5.22}$$

$$\mathcal{R}_{\mu\nu|\alpha\beta} = \gamma\left(\frac{1}{2}\left(\partial_\mu\mathcal{G}_{\alpha\beta|\nu} - \partial_\nu\mathcal{G}_{\alpha\beta|\mu} + \partial_\alpha\mathcal{G}_{\mu\nu|\beta} - \partial_\beta\mathcal{G}_{\mu\nu|\alpha}\right)\right), \tag{5.23}$$

and so will be all their derivatives, of order one and higher, such that we can state that there is no non-trivial object in $H(\gamma)$ involving the pure ghost number two ghosts $\{C_{\mu\nu}, \mathcal{C}_{\nu\alpha}\}$ and/or their derivatives. (An alternative manner of deriving the same result is to start with the transformed pure ghost number two ghost $\mathcal{C}'_{\nu\alpha}$ defined in (3.30) and with its derivatives. From the last relation in (3.32) we notice that the only γ-invariant linear elements in these quantities are $\partial_{[\mu}\mathcal{C}'_{\nu]\alpha}$, together with their derivatives. However, the second formula in (3.32) exhibits the γ-triviality of $\partial_{[\mu}\mathcal{C}'_{\nu]\alpha}$, such that all their subsequent derivatives will also be trivial in $H(\gamma)$. This ensures the fact that there is no non-trivial object in $H(\gamma)$ involving the pure ghost number two ghosts and/or their derivatives, since $\mathcal{C}'_{\nu\alpha}$ and its derivatives give all the information on the original pure ghost number two ghosts $\{C_{\mu\nu}, \mathcal{C}_{\nu\alpha}\}$ and their derivatives (the components of $\mathcal{C}'_{\nu\alpha}$ are exactly the independent components of both $C_{\mu\nu}$ and $\mathcal{C}_{\nu\alpha}$).)

Finally, we focus on the pure ghost number three ghost C_ν. The last definition in (3.20) indicates that it is γ-closed, while the first two definitions in the same formula (or, equivalently, the last one from (3.32)) exhibit that its first-order derivatives are γ-trivial

$$\partial_\mu C_\nu = \gamma\left(-\frac{1}{6}\left(\mathcal{C}_{\mu\nu} - 3C_{\mu\nu}\right)\right) = \gamma\left(-\frac{1}{6}\mathcal{C}'_{\mu\nu}\right), \tag{5.24}$$

and so will be all their subsequent derivatives. Therefore, the only non-trivial linear combination in $H(\gamma)$ constructed out of the ghost for ghost for ghost and its derivatives is the undifferentiated C_ν itself.

The above discussion on $H(\gamma)$ can be synthesized in the following array

$$
\begin{array}{|c|cc|}
\hline
\text{pgh} & \text{generator type} & \text{non-trivial objects in } H(\gamma) \\
\hline
0 & \begin{cases} \Pi^{*\Delta}, \partial\Pi^{*\Delta}, \cdots \\ t_{\lambda\mu\nu|\alpha}, \partial t_{\lambda\mu\nu|\alpha}, \cdots \end{cases} & \begin{cases} \Pi^{*\Delta}, \partial\Pi^{*\Delta}, \cdots \\ F_{\lambda\mu\nu\xi|\alpha\beta}, \partial F_{\lambda\mu\nu\xi|\alpha\beta}, \cdots \end{cases} \\
1 & \begin{cases} \eta_{\lambda\mu\nu}, \partial\eta_{\lambda\mu\nu}, \cdots \\ \mathcal{G}_{\mu\nu|\alpha}, \partial\mathcal{G}_{\mu\nu|\alpha}, \cdots \end{cases} & \mathcal{F}_{\lambda\mu\nu\alpha} = \partial_{[\lambda}\eta_{\mu\nu\alpha]}, \\
2 & \begin{cases} C_{\mu\nu}, \partial C_{\mu\nu}, \cdots \\ \mathcal{C}_{\nu\alpha}, \partial\mathcal{C}_{\nu\alpha}, \cdots \end{cases} & \text{none} \\
3 & C_{\nu}, \partial C_{\nu}, \cdots & C_{\nu} \\
\hline
\end{array}
\tag{5.25}
$$

It follows directly from these observations that the cohomology of γ is generated by the curvature tensor, the antifields, all their derivatives, as well as by $\mathcal{F}_{\lambda\mu\nu\alpha}$ and the undifferentiated ghost C_ν. According to the results exposed so far, we can state that the general local solution to the equation (5.6) is, up to trivial, γ-exact contributions, of the type

$$a_I = \alpha_I \left(\left[\Pi^{*\Delta}\right], \left[F_{\lambda\mu\nu\xi|\alpha\beta}\right] \right) \omega^I \left(C_\nu, \mathcal{F}_{\lambda\mu\nu\alpha} \right) , \qquad I > 0 , \tag{5.26}$$

where the notation $f([q])$ means that the function f depends on the variable q and its subsequent derivatives up to a finite number. In the above, ω^I are the elements of pure ghost number I (and obviously of antighost number zero) of a basis in the space of polynomials in $\mathcal{F}_{\lambda\mu\nu\alpha}$ and C_ν, which is finite-dimensional since all these variables anticommute. The objects α_I (obviously non-trivial in $H^0(\gamma)$) were taken to have a bounded number of derivatives, and therefore they are polynomials in the antifields $\Pi^{*\Delta}$, in the curvature tensor $F_{\lambda\mu\nu\xi|\alpha\beta}$, as well as in their derivatives. They are required to fulfill the property $\mathrm{agh}(\alpha_I) = I$ in order to ensure that the ghost number of a_I is equal to zero. Moreover, for a_I to be bosonic, as required by the general prescriptions of the BRST method, α_I and ω^I must have the same statistics, $\varepsilon(\alpha_I) = \varepsilon(\omega^I)$. Due to their γ-closedness

$$\gamma\alpha_I = 0 , \tag{5.27}$$

α_I will be called "invariant polynomials". At zero antighost number, the invariant polynomials are polynomials in the curvature $F_{\lambda\mu\nu\xi|\alpha\beta}$ and its derivatives. The result that we can replace the equation (5.3) with the less obvious one (5.6) is a nice consequence of the fact that the cohomology of the exterior space-time differential is trivial in the space of invariant polynomials at strictly positive antighost numbers. This means that if the invariant polynomial α_I of strictly positive antighost number is annihilated by d, then it can be written like the d-variation of precisely an invariant polynomial. For details, see the appendix A.

Replacing the solution (5.26) into the equation (5.4) and taking into account the definitions (3.22)–(3.26), we remark that a necessary (but not sufficient) condition for the existence of (non-trivial) solutions a_{I-1} is that the invariant polynomials α_I are (non-trivial) objects from the local cohomology of the Koszul-Tate differential $H(\delta|d)$ at antighost number $I > 0$ and pure ghost number equal to zero,[4] i.e.,

$$\delta\alpha_I = \partial_\mu \overset{(I-1)^\mu}{j}, \qquad \mathrm{agh}\left(\overset{(I-1)^\mu}{j}\right) = I-1 \geq 0, \qquad \mathrm{pgh}\left(\overset{(I-1)^\mu}{j}\right) = 0. \tag{5.28}$$

The above notation is generic, in the sense that α_I and $\overset{(I-1)^\mu}{j}$ may actually carry supplementary Lorentz indices. Consequently, we need to investigate some of the main properties of the local cohomology of the Koszul-Tate differential $H(\delta|d)$ at pure ghost number zero and strictly positive antighost numbers in order to fully determine the component a_I of highest antighost number from the first-order deformation. As we have discussed in section 2, the free model under study is a linear gauge theory of Cauchy order equal to four. In agreement with the general results from [28] (also see [12, 26, 27]), one can state that $H(\delta|d)$ (at pure ghost number zero) is trivial at antighost numbers strictly greater than its Cauchy order

$$H_I(\delta|d) = 0, \qquad I > 4. \tag{5.29}$$

Moreover, if the invariant polynomial α_I, with $\mathrm{agh}(\alpha_I) = I \geq 4$, is trivial in $H_I(\delta|d)$, then it can be taken to be trivial also in $H_I^{\mathrm{inv}}(\delta|d)$

$$\left(\alpha_I = \delta b_{I+1} + \partial_\mu \overset{(I)^\mu}{c}, \ \mathrm{agh}(\alpha_I) = I \geq 4\right) \quad \Rightarrow \quad \alpha_I = \delta\beta_{I+1} + \partial_\mu \overset{(I)^\mu}{\gamma}, \tag{5.30}$$

with β_{I+1} and $\overset{(I)^\mu}{\gamma}$ invariant polynomials. (An element of $H_I^{\mathrm{inv}}(\delta|d)$ is defined via an equation similar to (5.28), but with the corresponding current also an invariant polynomial.) The result (5.30) will be proved in the appendix B, theorem 3. This is important since together with (5.29) ensure that all the local cohomology of the Koszul-Tate differential in the space of invariant polynomials is trivial in antighost number strictly greater than four

$$H_I^{\mathrm{inv}}(\delta|d) = 0, \qquad I > 4. \tag{5.31}$$

Looking at the definitions (3.29) involving the transformed antifields (3.27), we can organize the non-trivial representatives of $H_I(\delta|d)$ (at pure ghost number equal to zero) and $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I\geq 2}$ in the following array

agh	non-trivial representatives spanning $H_I(\delta|d)$ and $H_I^{\mathrm{inv}}(\delta|d)$
$I > 4$	none
$I = 4$	$C^{*\nu}$
$I = 3$	$\mathcal{C}'^{*\nu\alpha}$
$I = 2$	$\mathcal{G}'^{*\mu\nu\vert\alpha}$

(5.32)

We remark that there is no non-trivial element in $\left(H_I(\delta|d)\right)_{I\geq 2}$ or $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I\geq 2}$ that effectively involves the curvature tensor $F_{\lambda\mu\nu\xi|\alpha\beta}$ and/or its derivatives, and the same stands for

[4] We recall that the local cohomology $H(\delta|d)$ is completely trivial at both strictly positive antighost *and* pure ghost numbers (for instance, see [28], theorem 5.4 and [29]). We will come back later to this result.

the quantities that are more than linear in the antifields and/or depend on their derivatives. In principle, one can construct from the above representatives other non-trivial invariant polynomials from $H(\delta|d)$ or $H^{\text{inv}}(\delta|d)$ that depend on the space-time co-ordinates. For instance, it can be checked by direct computation that $\mathcal{G}'^{*\mu\nu|\alpha} f_{\mu\nu\alpha\rho} x^{\rho}$, with $f_{\mu\nu\alpha\rho}$ a completely antisymmetric and constant tensor, is a non-trivial representative from both $H_2(\delta|d)$ and $H_2^{\text{inv}}(\delta|d)$. However, we will discard such candidates as they would break the Poincaré invariance of the deformations. In contrast to the groups $(H_I(\delta|d))_{I\geq 2}$ and $\left(H_I^{\text{inv}}(\delta|d)\right)_{I\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ at pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on $H(\delta|d)$ and $H^{\text{inv}}(\delta|d)$ at strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. Indeed, due to (5.31), it follows that we can successively eliminate all the pieces with $I>4$ from the non-integrated density of the first-order deformation by adding only trivial terms (see the appendix C), so we can take, without loss of non-trivial objects, the condition

$$I \leq 4 \tag{5.33}$$

in the decomposition (5.2). In addition, the last representative is of the form (5.26), where the invariant polynomial is necessarily a non-trivial object from $H_I^{\text{inv}}(\delta|d)$ for $I=2,3,4$, respectively from $H_1(\delta|d)$ for $I=1$.

5.2 The case $I=4$

Assuming that $I=4$, the development (5.2) becomes

$$a = a_0 + a_1 + a_2 + a_3 + a_4\,, \tag{5.34}$$

with a_4 $(\gamma a_4 = 0)$ a non-trivial quantity from $H^4(\gamma)$, and thus of the form (see (5.26))

$$a_4 = \alpha_4 \omega^4 \left(C_\nu, \mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_4\,, \tag{5.35}$$

and α_4 a non-trivial object from $H_4^{\text{inv}}(\delta|d)$. Taking into account the fact that the elements of pure ghost number four of a basis in the space of polynomials in $\mathcal{F}_{\lambda\mu\nu\alpha}$ and C_ν are spanned by

$$\omega^4 : \left(\mathcal{F}_{\lambda\mu\nu\alpha} C_\beta, \mathcal{F}_{\lambda\mu\nu\alpha} \mathcal{F}_{\lambda_1\mu_1\nu_1\alpha_1} \mathcal{F}_{\lambda_2\mu_2\nu_2\alpha_2} \mathcal{F}_{\lambda_3\mu_3\nu_3\alpha_3}\right), \tag{5.36}$$

with $\mathcal{F}_{\lambda\mu\nu\alpha}$ given in (5.11), and that the general (non-trivial) representative of $H_4^{\text{inv}}(\delta|d)$ is the undifferentiated antifield C_ρ^* (see (5.32)), we get that

$$\begin{aligned} a_4 = {} & C_\rho^* \Big(f_1^{\rho\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2\lambda_3\mu_3\nu_3\alpha_3} \mathcal{F}_{\lambda\mu\nu\alpha} \mathcal{F}_{\lambda_1\mu_1\nu_1\alpha_1} \mathcal{F}_{\lambda_2\mu_2\nu_2\alpha_2} \mathcal{F}_{\lambda_3\mu_3\nu_3\alpha_3} + \\ & + f_2^{\rho\lambda\mu\nu\alpha\beta} \mathcal{F}_{\lambda\mu\nu\alpha} C_\beta \Big) + \gamma b_4\,, \end{aligned} \tag{5.37}$$

where the coefficients of the type f must be non-derivative constants. At this point we ask that the deformed gauge theory preserves the Cauchy order of the uncoupled model, which enforces the requirement that the interacting lagrangian is of maximum order equal to two in the space-time derivatives of the tensor field $t_{\lambda\mu\nu|\alpha}$ at each order in the coupling constant. This assumption is quite natural as it enforces that the interacting model smoothly passes to the free one in the zero limit of the coupling constant. The terms quartic in the pure ghost number one combinations of $\mathcal{F}_{\lambda\mu\nu\alpha}$, if consistent, would lead to an interaction vertex in a_0 of order five in the space-time derivatives, which is forbidden by the above assumption, such that we will set the corresponding coefficient equal to zero

$$f_1^{\rho\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2\lambda_3\mu_3\nu_3\alpha_3} = 0\,, \tag{5.38}$$

which leaves us with

$$a_4 = f_2^{\rho\lambda\mu\nu\alpha\beta} C_\rho^* \mathcal{F}_{\lambda\mu\nu\alpha} C_\beta + \gamma b_4\,. \tag{5.39}$$

Recalling that we search manifestly covariant (and Poincaré-invariant) deformations and work in $D \geq 5$ space-time dimensions, only one independent possibility emerges

$$D = 6\,, \qquad f_2^{\rho\lambda\mu\nu\alpha\beta} = c\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\,, \tag{5.40}$$

with c an arbitrary real constant, such that we further obtain

$$a_4 = c\varepsilon^{\rho\lambda\mu\nu\alpha\beta} C_\rho^* \mathcal{F}_{\lambda\mu\nu\alpha} C_\beta\,. \tag{5.41}$$

(Without loss of generality, we set the γ-exact contribution from (5.39) equal to zero.) If consistent, such an a_4 would produce an a_0 that breaks the PT invariance. Let us show that the solution (5.41) is however not consistent, in the sense that it cannot be lifted to a_2. In view of this, we pass to finding the remaining terms from (5.34) as solutions to the equations (5.4)–(5.5) for $I = 4$

$$\delta a_4 + \gamma a_3 = \partial_\mu \overset{(3)}{m}{}^{\mu}\,, \qquad \delta a_3 + \gamma a_2 = \partial_\mu \overset{(2)}{m}{}^{\mu}\,, \tag{5.42}$$

$$\delta a_2 + \gamma a_1 = \partial_\mu \overset{(1)}{m}{}^{\mu}\,, \qquad \delta a_1 + \gamma a_0 = \partial_\mu \overset{(0)}{m}{}^{\mu}\,. \tag{5.43}$$

Inserting (5.41) in the former equation from (5.42) and using the last definition in (3.29) together with the results (5.18) and (5.24), we find that

$$a_3 = -c\varepsilon^{\rho\lambda\mu\nu\alpha\beta} \mathcal{C}'^{*\gamma}{}_{\rho} \left(8\left(\partial_\lambda t_{\mu\nu\alpha|\gamma}\right) C_\beta + \mathcal{F}_{\lambda\mu\nu\alpha} \mathcal{C}'_{\gamma\beta}\right)\,, \tag{5.44}$$

where $\mathcal{C}'_{\gamma\beta}$ can be read from (3.30). For solving the latter equation in (5.42), we firstly compute δa_3 with the help of (5.44) and the second definition in (3.29), and then manipulate the resulting expression on behalf of (5.18), (5.24) and the second formula from (3.32), obtaining in the end

$$\begin{aligned}
\delta a_3 = {} & \partial_\sigma \left(2c\varepsilon^{\rho\lambda\mu\nu\alpha\beta} \mathcal{G}'^{*\sigma\gamma|}{}_{\rho} \left(8\left(\partial_\lambda t_{\mu\nu\alpha|\gamma}\right) C_\beta + \mathcal{F}_{\lambda\mu\nu\alpha} \mathcal{C}'_{\gamma\beta}\right)\right) + \\
& + \gamma \left(-c\varepsilon^{\rho\lambda\mu\nu\alpha\beta} \mathcal{G}'^{*\sigma\gamma|}{}_{\rho} \left(\frac{4}{3}\left(\partial_\lambda t_{\mu\nu\alpha|[\sigma}\right) \mathcal{C}'_{\gamma]\beta} + \mathcal{F}_{\lambda\mu\nu\alpha} \mathcal{G}'_{\sigma\gamma|\beta}\right)\right) - \\
& - 2c\varepsilon^{\rho\lambda\mu\nu\alpha\beta} \mathcal{G}'^{*\sigma\gamma|}{}_{\rho} F_{\lambda\mu\nu\alpha|\sigma\gamma} C_\beta\,,
\end{aligned} \tag{5.45}$$

where $F_{\lambda\mu\nu\alpha|\sigma\gamma}$ is the curvature tensor (see (2.29)) and $\mathcal{G}'_{\sigma\gamma|\beta}$ was introduced in (3.30). Comparing the second equation from (5.42) with (5.45), we observe that a_3 of the form (5.44) leads to a consistent a_2 if and only if

$$-2c\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*\sigma\gamma|}{}_{\rho}F_{\lambda\mu\nu\alpha|\sigma\gamma}C_{\beta}=\gamma b_2+\partial_\sigma \overset{(2)}{w}{}^{\sigma}, \tag{5.46}$$

for some b_2 and $\overset{(2)}{w}{}^{\sigma}$ with the properties

$$\mathrm{agh}\left(b_2\right)=2=\mathrm{agh}\left(\overset{(2)}{w}{}^{\sigma}\right),\qquad \mathrm{pgh}\left(b_2\right)=2,\qquad \mathrm{pgh}\left(\overset{(2)}{w}{}^{\sigma}\right)=3. \tag{5.47}$$

However, this cannot happen unless

$$c=0, \tag{5.48}$$

since on the one hand the left-hand side of (5.46) is a non-trivial element from $H^3\left(\gamma\right)$ of antighost number two, where the role of the invariant polynomial is played by $-2c\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*\sigma\gamma|}{}_{\rho}F_{\lambda\mu\nu\alpha|\sigma\gamma}$, which implies $b_2=0$, and on the other hand it obviously does not reduce to a divergence, such that $\overset{(2)}{w}{}^{\sigma}=0$. But $b_2=0$ and $\overset{(2)}{w}{}^{\sigma}=0$ holding simultaneously in (5.46) lead to (5.48), and, in consequence, to

$$a_4=0. \tag{5.49}$$

In conclusion, under the assumption on the maximum derivative order of the interacting lagrangian being equal to two, the first-order deformation (5.2), whose component of maximum antighost number is subject to the condition (5.33), can only end at $I\leq 3$.

5.3 The case $I=3$

In the case where $I=3$ in (5.2), we can write that

$$a=a_0+a_1+a_2+a_3, \tag{5.50}$$

and, in agreement with the general discussion from the subsection 5.1, we can always redefine a_3 such as to pertain to the cohomological space $H\left(\gamma\right)$. Consequently, we have that (see (5.26))

$$a_3=\alpha_3\omega^3\left(C_\nu,\mathcal{F}_{\lambda\mu\nu\alpha}\right)+\gamma b_3. \tag{5.51}$$

The consistency of a at antighost number two (or, in other words, the existence of a_2 as solution to the equation $\delta a_3+\gamma a_2=\partial_\mu \overset{(2)}{m}{}^{\mu}$) requires that α_3 is a non-trivial element from $H_3^{\mathrm{inv}}\left(\delta|d\right)$. Since the elements of pure ghost number equal to three of a basis in the space of polynomials in $\mathcal{F}_{\lambda\mu\nu\alpha}$ and C_ν are spanned by

$$\omega^3:\left(C_\beta,\mathcal{F}_{\lambda\mu\nu\alpha}\mathcal{F}_{\lambda_1\mu_1\nu_1\alpha_1}\mathcal{F}_{\lambda_2\mu_2\nu_2\alpha_2}\right), \tag{5.52}$$

and the general non-trivial representative of $H_3^{\mathrm{inv}}\left(\delta|d\right)$ is $\mathcal{C}'^{*\nu\alpha}$ (see (5.32) for $I=3$), we get that

$$a_3=\mathcal{C}'^{*}_{\rho\sigma}\left(f_1^{\rho\sigma\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2}\mathcal{F}_{\lambda\mu\nu\alpha}\mathcal{F}_{\lambda_1\mu_1\nu_1\alpha_1}\mathcal{F}_{\lambda_2\mu_2\nu_2\alpha_2}+f_2^{\rho\sigma\beta}C_\beta\right)+\gamma b_3, \tag{5.53}$$

where the coefficients denoted by f must be non-derivative constants. If we require that the maximum derivative order of the interacting lagrangian is equal to two, we must set

$$f_1^{\rho\sigma\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2} = 0\,, \tag{5.54}$$

because otherwise the corresponding interaction terms from a_0 would be of order four in the space-time derivatives, and hence we obtain that

$$a_3 = f_2^{\rho\sigma\beta} \mathcal{C}'^{*}_{\rho\sigma} C_\beta + \gamma b_3\,. \tag{5.55}$$

Asking in addition that a_3 is Poincaré-invariant and that the space-time dimension fulfills $D \geq 5$, we are left only with the vanishing solution

$$f_2^{\rho\sigma\beta} = 0\,, \tag{5.56}$$

which further induces that

$$a_3 = 0\,. \tag{5.57}$$

In conclusion, the first-order deformation (5.2) cannot stop (in a non-trivial manner) either to $I = 3$.

5.4 The case $I = 2$

Supposing that the non-integrated density of the first-deformation ends at antighost number two

$$a = a_0 + a_1 + a_2\,, \tag{5.58}$$

its last component can again be redefined such as to belong to $H(\gamma)$, and thus

$$a_2 = \alpha_2 \omega^2 \left(\mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_2\,. \tag{5.59}$$

(The ghost C_ν does not enter $\omega^2\left(\mathcal{F}_{\lambda\mu\nu\alpha}\right)$ since its pure ghost number is equal to three, while $\mathrm{pgh}\left(\omega^2\right) = 2$). Moreover, a necessary condition for the consistency of (5.58) at antighost number one (the existence of a_1 as solution to the equation $\delta a_2 + \gamma a_1 = \partial_\mu \overset{(1)}{m}{}^\mu$) is that α_2 represents a non-trivial element from $H_2^{\mathrm{inv}}(\delta|d)$. Using the fact that the elements of pure ghost number equal to two of a basis in the space of polynomials in $\mathcal{F}_{\lambda\mu\nu\alpha}$ are spanned by

$$\omega^2 : \left(\mathcal{F}_{\lambda\mu\nu\alpha}\mathcal{F}_{\lambda_1\mu_1\nu_1\alpha_1}\right), \tag{5.60}$$

and that the general non-trivial representative of $H_2^{\mathrm{inv}}(\delta|d)$ is $\mathcal{G}'^{*\mu\nu|\alpha}$ (see (5.32) for $I = 3$), we find that

$$a_2 = f_1^{\rho\sigma\beta\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1} \mathcal{G}'^{*}_{\rho\sigma|\beta} \mathcal{F}_{\lambda\mu\nu\alpha}\mathcal{F}_{\lambda_1\mu_1\nu_1\alpha_1} + \gamma b_2\,, \tag{5.61}$$

with the components of f some non-derivative constants. At this stage we ask, just as we did before, that the derivative order of the interacting lagrangian a_0 is at most equal to two, which yields

$$f_1^{\rho\sigma\beta\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1} = 0\,, \tag{5.62}$$

because, if consistent, the component (5.61) would lead to an a_0 of derivative order equal to three. Removing now the γ-trivial component left in (5.61), we reach the conclusion that

$$a_2 = 0\,, \tag{5.63}$$

and therefore we can only have $I \leq 1$ in the expansion (5.2). The result (5.63) emphasizes that the original abelian gauge algebra is rigid under the deformation procedure (since there is no term simultaneously linear in the antighost number two antifields and quadratic in the ghosts of pure ghost number equal to one allowed to enter a_2), such that the self-interactions of the tensor field $t_{\lambda\mu\nu|\alpha}$ cannot be of type (iii) (see the beginning of section 4).

5.5 The case $I = 1$

For $I = 1$, the first-order deformation (5.2) becomes

$$a = a_0 + a_1\,, \tag{5.64}$$

with the last component ($\gamma a_1 = 0$) generically written like (see (5.26))

$$a_1 = \alpha_1\left(\left[t^{*\lambda\mu\nu|\alpha}\right], \left[F_{\lambda\mu\nu\xi|\alpha\beta}\right]\right)\omega^1\left(\mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_1\,. \tag{5.65}$$

The invariant polynomial α_1 is linear in the antifield $t^{*\lambda\mu\nu|\alpha}$ and its derivatives up to a finite order since these are the only objects of antighost number equal to one from the BRST algebra, while the element of pure ghost number equal to one of a basis in the space of polynomials in $\mathcal{F}_{\lambda\mu\nu\alpha}$ is

$$\omega^1 : \left(\mathcal{F}_{\lambda\mu\nu\alpha}\right). \tag{5.66}$$

We have mentioned in the subsection 5.1 that the necessary condition for a to be (non-trivially) consistent in this particular case is that α_1 is a (non-trivial) element from the group $H_1(\delta|d)$, which is infinite-dimensional. The difficult task of explicitly computing $H_1(\delta|d)$ can however be avoided if we recall in the first place the assumption on the maximum derivative order of a_0 being equal to two. This assumption is extremely useful as it forbids α_1 to depend on either the curvature tensor $F_{\lambda\mu\nu\xi|\alpha\beta}$ or its derivatives. Indeed, assuming that α_1 effectively involves the curvature, then the component from (5.65) with the minimum number of derivatives is linear in the undifferentiated antifield $t^{*\lambda\mu\nu|\alpha}$, in the undifferentiated curvature, as well as in the objects from (5.66), so it already contains three space-time derivatives and, if consistent, it would produce an a_0 with four derivatives. As a consequence, we have that

$$a_1 = \alpha_1^{\mathrm{lin}}\left(\left[t^{*\lambda\mu\nu|\alpha}\right]\right)\mathcal{F}_{\lambda\mu\nu\alpha} + \gamma b_1. \tag{5.67}$$

Moreover, α_1^{lin} is further restricted not to depend on the derivatives of $t^{*\lambda\mu\nu|\alpha}$. Indeed, one can always move the derivatives (by performing an integration by parts in the corresponding functional built from a_1 and by subsequently neglecting the resulting divergences) such as to act on $\mathcal{F}_{\lambda\mu\nu\alpha}$, in which case we arrive at trivial, γ-exact contributions (see (5.18)), that can safely be removed.

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The previous discussion allows us to state that the only possible candidate is found in $D=6$ dimensions[5]

$$a_1 \equiv a_1^{(D=6)} = c\sigma^{\alpha\beta}\varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'} t^*_{\lambda\mu\nu|\alpha}\mathcal{F}_{\lambda'\mu'\nu'\beta} + \gamma b_1^{(D=6)} . \tag{5.68}$$

Let us investigate the solutions to the equation

$$\delta a_1^{(D=6)} + \gamma a_0^{(D=6)} = \partial^\rho \overset{(0)}{m}_\rho \ . \tag{5.69}$$

In order to evaluate $\delta a_1^{(D=6)}$, we use the identity

$$\varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'}\sigma^{\alpha\beta} \overset{(1)}{T}_{\lambda\mu\nu|\alpha} \overset{(2)}{T}_{\lambda'\mu'\nu'\beta} = -\frac{3}{4}\varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'}\sigma^{\alpha\beta} \overset{(1)}{T}_{\lambda\mu\beta|\alpha} \overset{(2)}{T}_{\nu\lambda'\mu'\nu'} , \tag{5.70}$$

that holds for any $\overset{(1)}{T}_{\lambda\mu\nu|\alpha}$ completely antisymmetric in its first three indices and for any completely antisymmetric $\overset{(2)}{T}_{\lambda'\mu'\nu'\beta}$, together with the first definition in (3.23) and the relation (2.23). Further calculations, based on the formula (3.17) and on the result (5.18), produce

$$\begin{aligned}\delta a_1^{(D=6)} &= \gamma\left(\frac{c}{2}(4-D)\,\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'} t_{\lambda\mu(\rho|\alpha)}\partial_{\lambda'}\left(\sigma^{\alpha\rho}\partial^\beta t_{\mu'\nu'\rho'|\beta} - \partial^\alpha t_{\mu'\nu'\rho'|}{}^{\rho}\right)\right) + \\ &\quad +\partial^\rho\left(\frac{3c}{4}(4-D)\,\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'}\left(\left(\partial_\rho t_{\lambda\mu} + 2\partial_\lambda t_{\mu\rho} - \partial^\alpha t_{\lambda\mu\rho|\alpha}\right)\mathcal{F}_{\lambda'\mu'\nu'\rho'} - \right.\right.\\ &\qquad\qquad \left.\left. - t_{\lambda\mu}\partial_\rho \mathcal{F}_{\lambda'\mu'\nu'\rho'} + \left(t_{\lambda\mu\nu|\rho} - 2\sigma_{\lambda\rho}t_{\mu\nu}\right)\partial^\nu \mathcal{F}_{\lambda'\mu'\nu'\rho'}\right)\right) - \\ &\quad -\frac{c}{2}(4-D)\,\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'} \mathcal{T}_{\lambda\mu(\rho|\alpha)}\partial_{\lambda'}\left(\sigma^{\alpha\rho}\partial^\beta t_{\mu'\nu'\rho'|\beta} - \partial^\alpha t_{\mu'\nu'\rho'|}{}^{\rho}\right) . \end{aligned} \tag{5.71}$$

Comparing (5.71) with (5.69), we have that the existence of $a_0^{(D=6)}$ requires that the last term in the right-hand side of (5.71) either vanishes or reduces to a divergence. Nevertheless, the second relation from (5.11) ensures that this term is not a divergence. Therefore it must vanish, which holds if and only if

$$c = 0 , \tag{5.72}$$

so it yields

$$a_1^{(D=6)} = 0 . \tag{5.73}$$

We actually showed that

$$a_1 = 0 , \tag{5.74}$$

and therefore the first-order deformation (5.2) may only reduce to its antighost number zero component ($I=0$). Since the antighost number one terms from the solution to the master equation control the gauge symmetries of a given gauge theory, the result (5.74) tells us that there are no consistent self-interactions of the tensor field $t_{\lambda\mu\nu|\alpha}$ that deform the gauge transformations (2.8), or, which is the same, there are no type (ii) interactions (see the classification given at the beginning of section 4).

[5]Apparently, there are more possibilities: one is of the type $t^{*\lambda\mu\nu|\alpha}\mathcal{F}_{\lambda\mu\nu\alpha} = \frac{1}{4}\, t^{*[\lambda\mu\nu|\alpha]}\mathcal{F}_{\lambda\mu\nu\alpha}$, but it vanishes identically due to (3.3), while those involving Levi-Civita symbols contracted on all the indices of $t^{*\lambda\mu\nu|\alpha}$ vanish for the same reason.

5.6 The case $I = 0$

Thus, we remain only with the possibility that the non-integrated density of the first-order deformation (5.2) coincides with the deformed lagrangian at the first order in the coupling constant

$$a = a_0 \left(\left[t_{\lambda\mu\nu|\alpha} \right] \right) , \tag{5.75}$$

which is subject to the equation

$$\gamma a_0 = \partial_\mu \overset{(0)}{m}{}^{\mu} . \tag{5.76}$$

There are two main types of solutions to the last equation. The first one corresponds to $\overset{(0)}{m}{}^{\mu} = 0$ and is given by functions in the field $t_{\lambda\mu\nu|\alpha}$ and its derivatives that are invariant under the gauge transformations (2.8). As the components of the curvature tensor (2.29) are the most general gauge-invariant objects, it follows that

$$\gamma a_0' = 0 \quad \Rightarrow \quad a_0' = a_0' \left(\left[F_{\lambda\mu\nu\xi|\alpha\beta} \right] \right) . \tag{5.77}$$

At this point we recall the requirement that the interacting lagrangian is of maximum order equal to two in the space-time derivatives of the tensor field, which leads to

$$a_0' = 0 , \tag{5.78}$$

because otherwise a_0' would be at least quadratic in the curvature tensor, and thus of minimum derivative order equal to four. The second type of solutions is associated with $\overset{(0)}{m}{}^{\mu} \neq 0$, being understood that we maintain the restriction on the derivative order of a_0 and discard the divergence-like solutions $a_0 = \partial_\mu u^\mu$. Denoting the Euler-Lagrange derivatives of a_0 by $A^{\lambda\mu\nu|\alpha} \equiv \delta a_0 / \delta t_{\lambda\mu\nu|\alpha}$, the equation (5.76) implies that the tensor $A^{\lambda\mu\nu|\alpha}$, which has the same mixed symmetry like $t_{\lambda\mu\nu|\alpha}$ (it is antisymmetric in its first three indices and fulfills the identity $A^{[\lambda\mu\nu|\alpha]} \equiv 0$), is subject to the equations

$$\partial_\lambda A^{\lambda\mu\nu|\alpha} = 0 , \qquad \partial_\alpha A^{\lambda\mu\nu|\alpha} = 0 . \tag{5.79}$$

According to the discussion from the end of the section 2 (see the formulas (2.45)–(2.46)), the general solution of (5.79) is

$$\frac{\delta a_0}{\delta t_{\lambda\mu\nu|\alpha}} \equiv A^{\lambda\mu\nu|\alpha} = \partial_\xi \partial_\beta \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta} , \tag{5.80}$$

where $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ has the mixed symmetry (2.30) of the curvature tensor. Imposing that $A^{\lambda\mu\nu|\alpha}$ contains at most two derivatives, we find that $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ involves only the undifferentiated tensor field $t_{\lambda\mu\nu|\alpha}$. Let N be a derivation in the algebra of the fields $t_{\lambda\mu\nu|\alpha}$ and of their derivatives that counts the powers of the fields and their derivatives, defined by

$$N = \sum_{n \geq 0} \left(\partial_{\mu_1} \cdots \partial_{\mu_n} t_{\lambda\mu\nu|\alpha} \right) \frac{\partial}{\partial \left(\partial_{\mu_1} \cdots \partial_{\mu_n} t_{\lambda\mu\nu|\alpha} \right)} . \tag{5.81}$$

Then, it is easy to see that for every non-integrated density u, we have that

$$N u = t_{\lambda\mu\nu|\alpha} \frac{\delta u}{\delta t_{\lambda\mu\nu|\alpha}} + \partial_\mu s^\mu , \tag{5.82}$$

where $\delta u/\delta t_{\lambda\mu\nu|\alpha}$ denotes the variational derivative of u. If u is an homogeneous polynomial of order $p>0$ in the fields and their derivatives, then $Nu = pu$, such that

$$u = \frac{1}{p} t_{\lambda\mu\nu|\alpha} \frac{\delta u}{\delta t_{\lambda\mu\nu|\alpha}} + \partial_\mu \left(\frac{1}{p} s^\mu\right). \tag{5.83}$$

As a_0 can always be decomposed as a sum of homogeneous polynomials of various orders, it is enough to analyze the equation (5.76) for a fixed value of p. Setting $u = a_0$ in (5.83) and using (5.80), we find that

$$a_0 = \frac{1}{p} t_{\lambda\mu\nu|\alpha} \partial_\xi \partial_\beta \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta} + \partial_\mu \tilde{s}^\mu . \tag{5.84}$$

Integrating twice by parts in (5.84) and taking into account the mixed symmetry of $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$, we infer that

$$a_0 = k F_{\lambda\mu\nu\xi|\alpha\beta} \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta} + \partial_\mu l^\mu , \tag{5.85}$$

with $k = 1/8p$. Acting now with γ on (5.85), we obtain that

$$\begin{aligned} \gamma a_0 = & -3k\eta_{\gamma\delta\tau} \partial_\sigma \left(F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}} + F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\sigma\delta\tau|\gamma}} \right) - \\ & -3k\mathcal{G}_{\delta\tau|\sigma} \partial_\gamma \left(F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}} \right) + \partial_\mu \bar{l}^\mu , \end{aligned} \tag{5.86}$$

for some $\bar{l}^\mu$. From (5.86) we observe that a_0 satisfies the equation (5.76) if and only if

$$\partial_\sigma \left(F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}} \right) = 0 , \qquad \partial_\gamma \left(F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}} \right) = 0 . \tag{5.87}$$

Since the quantity between parentheses in (5.87) has the same mixed symmetry like the tensor field $t_{\gamma\delta\tau|\sigma}$, with the help of the relations (2.45)–(2.46) we determine that

$$F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}} = \partial_\rho \partial_\theta \psi^{\gamma\delta\tau\rho|\sigma\theta} , \tag{5.88}$$

for some $\psi^{\gamma\delta\tau\rho|\sigma\theta}$ with the mixed symmetry of the curvature tensor, that depends on the undifferentiated tensor field $t_{\lambda\mu\nu|\alpha}$. Computing the left-hand side of (5.88), we arrive at

$$\begin{aligned} -F_{\lambda\mu\nu\xi|\alpha\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}} = & \ \partial_\rho \partial_\theta \left(-8 t_{\lambda\mu\nu|\beta} \frac{\partial \tilde{\Phi}^{\lambda\mu\nu\rho|\beta\theta}}{\partial t_{\gamma\delta\tau|\sigma}} \right) + \frac{\partial^2 \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma} \partial t_{\gamma'\delta'\tau'|\sigma'}} \times \\ & \times \left(\left(\partial^\theta t_{\gamma'\delta'\tau'|\sigma'}\right) \partial^\rho \left(\sigma_{\theta\alpha}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\beta} - \sigma_{\theta\beta}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\alpha}\right) + \right. \\ & + \left(\partial^\rho t_{\gamma'\delta'\tau'|\sigma'}\right) \partial^\theta \left(\sigma_{\theta\alpha}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\beta} - \sigma_{\theta\beta}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\alpha}\right) + \\ & \left. + \left(\sigma_{\theta\alpha}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\beta} - \sigma_{\theta\beta}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\alpha}\right) \partial^\rho \partial^\theta t_{\gamma'\delta'\tau'|\sigma'} \right) + \\ & + \frac{\partial^3 \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma} \partial t_{\gamma'\delta'\tau'|\sigma'} \partial t_{\gamma''\delta''\tau''|\sigma''}} \left(\sigma_{\theta\alpha}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\beta} - \sigma_{\theta\beta}\sigma_{\rho[\lambda} t_{\mu\nu\xi]|\alpha}\right) \times \\ & \times \left(\partial^\theta t_{\gamma'\delta'\tau'|\sigma'}\right) \left(\partial^\rho t_{\gamma''\delta''\tau''|\sigma''}\right) . \end{aligned} \tag{5.89}$$

The right-hand side of (5.89) can be written in the form of the right-hand side of (5.88) if and only if

$$8t_{\lambda\mu\nu|\beta}\frac{\partial\tilde{\Phi}^{\lambda\mu\nu\rho|\beta\theta}}{\partial t_{\gamma\delta\tau|\sigma}}=\psi^{\gamma\delta\tau\rho|\sigma\theta}\,, \tag{5.90}$$

$$\frac{\partial^2\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}\partial t_{\gamma'\delta'\tau'|\sigma'}}=0\,,\qquad \frac{\partial^3\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}}{\partial t_{\gamma\delta\tau|\sigma}\partial t_{\gamma'\delta'\tau'|\sigma'}\partial t_{\gamma''\delta''\tau''|\sigma''}}=0\,. \tag{5.91}$$

On the one hand, the requirement (5.91) restricts $\tilde{\Phi}^{\lambda\mu\nu\rho|\beta\theta}$ to be linear in $t_{\lambda\mu\nu|\alpha}$ and, on the other hand, we have the condition that $\tilde{\Phi}^{\lambda\mu\nu\rho|\beta\theta}$ has the same mixed symmetry like the curvature tensor. These considerations fix $\tilde{\Phi}^{\lambda\mu\nu\rho|\beta\theta}$ to be precisely of the type

$$\tilde{\Phi}^{\lambda\mu\nu\rho|\beta\theta}=k'\Phi^{\lambda\mu\nu\rho|\beta\theta}\,, \tag{5.92}$$

where k' is a real constant and $\Phi^{\lambda\mu\nu\rho|\beta\theta}$ is the tensor involved in the functions (2.47) that yield the free field equations, being expressed by (2.48). Meanwhile, (5.92) fixes the value of p from (5.84) to $p=2$. By direct computation we deduce that (5.90) is also satisfied, and get that

$$\psi^{\gamma\delta\tau\rho|\sigma\theta}=-8k'\Phi^{\gamma\delta\tau\rho|\sigma\theta}\,. \tag{5.93}$$

Inserting (5.92) in (5.84) (and taking $p=2$), we finally infer that

$$a_0\left(\left[t_{\lambda\mu\nu|\alpha}\right]\right)=\frac{k'}{2}t_{\lambda\mu\nu|\alpha}T^{\lambda\mu\nu|\alpha}+\partial_\mu l^\mu\,, \tag{5.94}$$

hence (5.94) is (up to an irrelevant divergence) proportional with the original lagrangian. This solution is however trivial in $H^0\left(s|d\right)$ since it can be expressed like

$$a_0=sb+\partial_\mu v^\mu\,,\qquad \mathrm{gh}\left(b\right)=-1\,,\qquad \mathrm{gh}\left(v^\mu\right)=0\,, \tag{5.95}$$

where

$$b=\frac{k'}{2}\left(t^{*\lambda\mu\nu|\alpha}t_{\lambda\mu\nu|\alpha}+\mathcal{G}'^{*\mu\nu|\alpha}\mathcal{G}'_{\mu\nu|\alpha}+\mathcal{C}'^{*\nu\alpha}\mathcal{C}'_{\nu\alpha}+C^{*\alpha}C_\alpha\right), \tag{5.96}$$

$$v^\mu=\left(l^\mu+3k'\left(\mathcal{C}'^{*\mu\alpha}C_\alpha-\frac{1}{2}t^{*\mu\nu\rho|\alpha}\mathcal{G}'_{\nu\rho|\alpha}\right)-k'\mathcal{G}'^{*\mu\nu|\alpha}\mathcal{C}'_{\nu\alpha}\right). \tag{5.97}$$

Then, in agreement with the discussion from the beginning of this section, the solution (5.94) can be safely removed from the first-order deformation by replacing it with

$$a_0\left(\left[t_{\lambda\mu\nu|\alpha}\right]\right)=0, \tag{5.98}$$

and hence there are no non-trivial interactions, invariant under the original gauge symmetries (2.8), that can be added to the action (2.1) (type (i) interactions).

The results (5.49), (5.57), (5.63), (5.74) and (5.98) allow us to write that

$$S_1=0\,, \tag{5.99}$$

which further yields that we can set

$$S_k=0\,,\qquad k>1\,, \tag{5.100}$$

as solutions to the higher-order deformation equations (4.5), etc. The general conclusion of this section is that, under the assumptions of Lorentz covariance, Poincaré invariance and space-time locality of the deformations, combined with the requirement on the maximum derivative order of the interacting lagrangian to be equal to two, there are no consistent and non-trivial self-interactions for the mixed symmetry-type tensor $t_{\lambda\mu\nu|\alpha}$, described in $D \geq 5$ space-time dimensions by the action (2.1), subject to the gauge symmetries (2.8).

6. No consistent cross-interactions with the Pauli-Fierz field

The next aim of this paper is to investigate the interactions between the tensor field $t_{\lambda\mu\nu|\alpha}$ and a non-interacting massless spin-2 field $h_{\mu\nu}$, described by the Pauli-Fierz action [20]. The self-interactions of a single massless spin-2 field have been extensively studied in the literature and are known to lead to the Einstein-Hilbert action with a cosmological term. A recent cohomological treatment of a collection of non-interacting massless spin-2 fields [27] confirms this result and, moreover, establishes that there are no consistent cross-couplings in multi-graviton theories. In fact, it is proved that the only possible deformations are given by a sum of individual Einstein-Hilbert actions, one for each spin-two field in the collection. In this section we will mainly focus on the cross-couplings, i.e., on the interactions that mix the fields $t_{\lambda\mu\nu|\alpha}$ and $h_{\mu\nu}$, and will not insist on the cohomological construction of the Einstein-Hilbert action with a cosmological term, which can be found in detail in [27].

6.1 Free model and BRST symmetry

We start from a free action, written as the sum between (2.1) and the Pauli-Fierz action in $D \geq 5$ space-time dimensions

$$S_0\left[t_{\lambda\mu\nu|\alpha}, h_{\mu\nu}\right] = S_0\left[t_{\lambda\mu\nu|\alpha}\right] + S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right], \tag{6.1}$$

with

$$\begin{aligned} S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right] = \int d^D x \Big(& -\frac{1}{2}\left(\partial^\rho h^{\mu\nu}\right)\left(\partial_\rho h_{\mu\nu}\right) + \left(\partial_\rho h^{\rho\mu}\right)\left(\partial^\lambda h_{\lambda\mu}\right) - \\ & - \left(\partial^\rho h\right)\left(\partial^\lambda h_{\lambda\rho}\right) + \frac{1}{2}\left(\partial^\rho h\right)\left(\partial_\rho h\right)\Big), \end{aligned} \tag{6.2}$$

where $h_{\mu\nu}$ is symmetric and h denotes its trace. Action (6.2) is invariant under the abelian and irreducible gauge transformations

$$\delta_\epsilon h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)}\,. \tag{6.3}$$

The presence of the gauge transformations (6.3) shows that the functions that define the field equations of the Pauli-Fierz action

$$\frac{\delta S_0^{\mathrm{PF}}}{\delta h_{\mu\nu}} \equiv -2H^{\mu\nu} \approx 0\,, \tag{6.4}$$

are not all independent, but satisfy the Noether identities

$$\partial_\mu H^{\mu\nu} = 0\,. \tag{6.5}$$

In the above, $H^{\mu\nu}$ represents the linearized Einstein tensor

$$\begin{aligned} H^{\mu\nu} &= K^{\mu\nu} - \frac{1}{2}\sigma^{\mu\nu} K \\ &= -\frac{1}{2}\left(\Box h^{\mu\nu} + \partial^{\mu}\partial^{\nu} h - \partial_{\rho}\partial^{(\mu} h^{\nu)\rho} + \sigma^{\mu\nu}\left(\partial_{\alpha}\partial_{\beta} h^{\alpha\beta} - \Box h\right)\right), \qquad (6.6) \\ H^{\mu\nu} &= H^{\nu\mu}, \qquad (6.7) \end{aligned}$$

with $K^{\mu\nu}$ the linearized Ricci tensor and K the linearized scalar curvature, which are defined with the help of the linearized Riemann tensor

$$K^{\mu\nu|\alpha\beta} = -\frac{1}{2}\left(\partial^{\mu}\partial^{\alpha} h^{\nu\beta} - \partial^{\nu}\partial^{\alpha} h^{\mu\beta} - \partial^{\mu}\partial^{\beta} h^{\nu\alpha} + \partial^{\nu}\partial^{\beta} h^{\mu\alpha}\right), \qquad (6.8)$$

via its simple, respectively, double trace

$$\begin{aligned} K^{\mu\nu} &= K_{\alpha}{}^{\mu|\alpha\nu} = -\frac{1}{2}\left(\Box h^{\mu\nu} + \partial^{\mu}\partial^{\nu} h - \partial_{\rho}\partial^{(\mu} h^{\nu)\rho}\right), \qquad (6.9) \\ K &= K_{\mu}{}^{\mu} = -\Box h + \partial_{\alpha}\partial_{\beta} h^{\alpha\beta}. \qquad (6.10) \end{aligned}$$

The linearized Riemann tensor $K^{\mu\nu|\alpha\beta}$ has the mixed symmetry of the rectangular two-column Young diagram

$$K^{\mu\nu|\alpha\beta} = \begin{array}{|c|c|} \hline \mu & \alpha \\ \hline \nu & \beta \\ \hline \end{array}\,, \qquad (6.11)$$

so it is separately antisymmetric in the pairs $\mu\nu$ and $\alpha\beta$

$$K^{\mu\nu|\alpha\beta} = -K^{\nu\mu|\alpha\beta} = -K^{\mu\nu|\beta\alpha}, \qquad (6.12)$$

is symmetric under the permutation of these pairs ($\mu\nu \longleftrightarrow \alpha\beta$),

$$K^{\mu\nu|\alpha\beta} = K^{\alpha\beta|\mu\nu}, \qquad (6.13)$$

and satisfies the (algebraic) Bianchi I identity

$$K^{[\mu\nu|\alpha]\beta} = 0. \qquad (6.14)$$

In addition, it also fulfills the linearized (differential) Bianchi II identity

$$\partial^{[\lambda} K^{\mu\nu]|\alpha\beta} = 0. \qquad (6.15)$$

The most general gauge-invariant objects that can be constructed out of $h_{\mu\nu}$ are the linearized Riemann tensor and its space-time derivatives. The Pauli-Fierz action alone underlies a linear, free gauge theory of Cauchy order equal to two. Taking into account the fact that the Cauchy order of the theory describing the tensor field $t_{\lambda\mu\nu|\alpha}$ is equal to four and that the action (6.1) does not couple $t_{\lambda\mu\nu|\alpha}$ to the Pauli-Fierz field, we obtain that (6.1) is a linear, non-interacting gauge theory of Cauchy order equal to four.

The main features of the Pauli-Fierz theory can be understood in an elegant fashion via the generalized differential complex $\Omega_2(\mathcal{M})$ of tensor fields with mixed symmetries corresponding to a *maximal sequence* of two-column Young diagrams, defined on a pseudo-riemannian manifold $\mathcal{M}$ of dimension D. For details, see for instance [8] and [25]. We only recall here some basic results. If we denote by $\bar{d}$ the associated operator (3-differential), which is third-order nilpotent, $\bar{d}^3 = 0$, then we observe that linearized Riemann tensor is proportional with $\bar{d}^2$ acting on the Pauli-Fierz field $h_{\mu\nu}$

$$K_{\mu\nu|\alpha\beta} \sim \left(\bar{d}^2 h\right)_{\mu\nu\alpha\beta}, \qquad h_{\mu\nu}, K_{\mu\nu|\alpha\beta} \in \Omega_2\left(\mathcal{M}\right). \tag{6.16}$$

Therefore, the Bianchi II identity (6.15) is nothing but a consequence of $\bar{d}^3 = 0$ on $\Omega_2\left(\mathcal{M}\right)$

$$\left(\bar{d}K\right)_{\lambda\mu\nu\alpha\beta} = \left(\bar{d}^3 h\right)_{\lambda\mu\nu\alpha\beta} \sim \partial_{[\lambda} K_{\mu\nu]|\alpha\beta} \equiv 0 . \tag{6.17}$$

There appear interesting results if we refer to the generalized cohomology of $\bar{d}$ on $\Omega_2\left(\mathcal{M}\right)$, where $\mathcal{M}$ has the trivial topology of $\mathbb{R}^D$, combined with the operation of generalized Hodge duality. For example, let us consider a symmetric covariant tensor field $\bar{H}^{\mu\nu}$, which is subject to the equation

$$\partial_\mu \bar{H}^{\mu\nu} = 0 . \tag{6.18}$$

Then, there exists a tensor $\bar{\Phi}^{\mu\alpha|\nu\beta}$ with the mixed symmetry of the linearized Riemann tensor, such that

$$\bar{H}^{\mu\nu} = \partial_\alpha \partial_\beta \bar{\Phi}^{\mu\alpha|\nu\beta} . \tag{6.19}$$

The above statement can be easily verified with respect to the linearized Einstein tensor (6.6), which satisfies the Noether identity (6.5) and can indeed be written in the form

$$H^{\mu\nu} = \partial_\alpha \partial_\beta \Phi^{\mu\alpha|\nu\beta} , \tag{6.20}$$

where the corresponding $\Phi^{\mu\alpha|\nu\beta}$ reads as

$$\begin{aligned}\Phi^{\mu\alpha|\nu\beta} = \frac{1}{2}\Big(& -h^{\mu\nu}\sigma^{\alpha\beta} + h^{\alpha\nu}\sigma^{\mu\beta} + h^{\mu\beta}\sigma^{\alpha\nu} - \\ & - h^{\alpha\beta}\sigma^{\mu\nu} + \left(\sigma^{\mu\nu}\sigma^{\alpha\beta} - \sigma^{\alpha\nu}\sigma^{\mu\beta}\right) h\Big).\end{aligned} \tag{6.21}$$

The BRST complex for the free theory (6.1), subject to the gauge symmetries (2.8) and (6.3), comprises the BRST generators introduced in section 3 and associated with the theory (2.1), as well as the Pauli-Fierz field $h_{\mu\nu}$, the fermionic ghost η_μ corresponding to the gauge invariance (6.3), together with the antifields $h^{*\mu\nu}$ and $\eta^{*\mu}$ from the Pauli-Fierz sector. The BRST differential of the entire free gauge theory splits like in (3.7) and its components obey again the nilpotency and anticommutation relations (3.16). The actions of γ and δ on the BRST generators corresponding to the tensor field $t_{\lambda\mu\nu|\alpha}$ are expressed by the formulas (3.17)–(3.26). [Also see the transformed ghosts/antifields (3.27) and (3.30), together with the accompanying formulas (3.29) and (3.32).] Related to the Pauli-Fierz sector, γ and δ act via the definitions

$$\gamma h_{\mu\nu} = \partial_{(\mu} \eta_{\nu)} , \qquad \gamma \eta_\mu = 0 , \tag{6.22}$$

$$\gamma h^{*\mu\nu} = 0 = \gamma \eta^{*\mu} , \tag{6.23}$$

$$\delta h_{\mu\nu} = 0 = \delta \eta_\mu , \tag{6.24}$$

$$\delta h^{*\mu\nu} = 2H_{\mu\nu} , \qquad \delta \eta^{*\mu} = -2\partial_\nu h^{*\nu\mu} . \tag{6.25}$$

The pure ghost number and antighost number of the BRST generators corresponding to the tensor field $t_{\lambda\mu\nu|\alpha}$ can be found in (3.8)–(3.15) (see also (3.28) and (3.31)), while for the Pauli-Fierz field/ghost/antifield sector are given below

$$\mathrm{pgh}\left(h_{\mu\nu}\right) = 0\,, \qquad \mathrm{pgh}\left(\eta_{\mu}\right) = 1\,, \qquad \mathrm{pgh}\left(h^{*\mu\nu}\right) = 0 = \mathrm{pgh}\left(\eta^{*\mu}\right), \tag{6.26}$$

$$\mathrm{agh}\left(h_{\mu\nu}\right) = 0 = \mathrm{agh}\left(\eta_{\mu}\right), \qquad \mathrm{agh}\left(h^{*\mu\nu}\right) = 1\,, \qquad \mathrm{agh}\left(\eta^{*\mu}\right) = 2\,. \tag{6.27}$$

In agreement with the general line of the antifield-BRST method, it results that the free BRST differential s for the theory (6.1) is canonically generated in the antibracket ($s\cdot = (\cdot, S)$) by the solution to the master equation $(S,S) = 0$, which in our case has the form

$$S = S^{\mathrm{t}} + S^{\mathrm{h}}\,, \tag{6.28}$$

where S^{t} is given by the right-hand side of (3.34) and

$$S^{\mathrm{h}} = S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right] + \int d^D x\, h^{*\mu\nu}\partial_{(\mu}\eta_{\nu)}\,. \tag{6.29}$$

It is understood that the present antibracket takes into account all the BRST generators, including those from the Pauli-Fierz sector.

6.2 $H(\gamma)$ and $H(\delta|d)$

In order to determine the solution to the local first-order deformation equation (4.4), we proceed like in section 5, namely, we expand the non-integrated density according to the antighost number as in (5.2) and solve the equivalent tower of equations, given by (5.6) and (5.4)–(5.5). It is convenient to split the first-order deformation into

$$a = a^{\mathrm{h-h}} + a^{\mathrm{h-t}}\,, \tag{6.30}$$

where $a^{\mathrm{h-h}}$ denotes the part responsible for the self-interactions of the Pauli-Fierz field and $a^{\mathrm{h-t}}$ signifies the component that describes only the cross-interactions between $h_{\mu\nu}$ and $t_{\lambda\mu\nu|\alpha}$. (We have seen in the previous section that we can take the component related to the self-interactions of the tensor field $t_{\lambda\mu\nu|\alpha}$ to be trivial, so we already discarded it from (6.30).) Then, $a^{\mathrm{h-h}}$ is completely known (for a detailed analysis, see for instance [27])

$$a^{\mathrm{h-h}} = a_0^{\mathrm{h-h}} + a_1^{\mathrm{h-h}} + a_2^{\mathrm{h-h}}\,, \tag{6.31}$$

where

$$a_2^{\mathrm{h-h}} = \eta^{*\mu}\eta^{\alpha}\partial_{\mu}\eta_{\alpha}\,, \tag{6.32}$$

$$a_1^{\mathrm{h-h}} = -h^{*\mu\nu}\eta^{\alpha}\left(\partial_{\mu}h_{\nu\alpha} + \partial_{\nu}h_{\mu\alpha} - \partial_{\alpha}h_{\mu\nu}\right), \tag{6.33}$$

and $a_0^{\mathrm{h-h}}$ is the cubic vertex of the Einstein-Hilbert lagrangian plus a cosmological term. Inserting (6.30) in (5.2) and using the fact that the first component already obeys the equation

$$sa^{\mathrm{h-h}} = \partial_{\mu}u^{\mu}\,, \tag{6.34}$$

it follows that only $a^{\mathrm{h-t}}$ is unknown, being subject to the equation

$$sa^{\mathrm{h-t}} = \partial_\mu w^\mu \,. \tag{6.35}$$

If we develop $a^{\mathrm{h-t}}$ according to the antighost number[6]

$$a^{\mathrm{h-t}} = \sum_{k=0}^{I} a_k^{\mathrm{h-t}}, \qquad \mathrm{agh}\left(a_k^{\mathrm{h-t}}\right) = k\,, \qquad \mathrm{gh}\left(a_k^{\mathrm{h-t}}\right) = 0\,, \qquad \varepsilon\left(a_k^{\mathrm{h-t}}\right) = 0, \tag{6.36}$$

then (6.35) is equivalent to the tower of equations[7]

$$\gamma a_I^{\mathrm{h-t}} = 0\,, \tag{6.37}$$

$$\delta a_I^{\mathrm{h-t}} + \gamma a_{I-1}^{\mathrm{h-t}} = \partial_\mu \overset{(I-1)^\mu}{w}\,, \tag{6.38}$$

$$\delta a_k^{\mathrm{h-t}} + \gamma a_{k-1}^{\mathrm{h-t}} = \partial_\mu \overset{(k-1)^\mu}{w}\,, \qquad I-1 \geq k \geq 1\,, \tag{6.39}$$

where $\left(\overset{(k)^\mu}{w}\right)_{k=\overline{0,I}}$ are some local currents, with $\mathrm{agh}\left(\overset{(k)^\mu}{w}\right) = k$. We recall that due to (6.35) $a^{\mathrm{h-t}}$ belongs to the cohomological space $H^0(s|d)$, so on the one hand it is defined up to s-exact modulo d objects, i.e., we can always replace $a^{\mathrm{h-t}}$ with $a^{\mathrm{h-t}} + sb + \partial_\mu t^\mu$, $\mathrm{gh}(b) = -1$, $\mathrm{gh}(t^\mu) = 0$, and on the other hand we can always discard the purely trivial deformations, namely, if $a^{\mathrm{h-t}} = sb + \partial_\mu t^\mu$, then we can safely set $a^{\mathrm{h-t}} = 0$.

The equation (6.37) shows that $a_I^{\mathrm{h-t}} \in H(\gamma)$, such that its solution is unique up to trivial (γ-exact) contributions, $a_I^{\mathrm{h-t}} \rightarrow a_I^{\mathrm{h-t}} + \gamma b_I$, while any purely γ-exact solution $a_I^{\mathrm{h-t}} = \gamma b_I$ can be taken to vanish, $a_I^{\mathrm{h-t}} = 0$. In order to infer the general solution to this equation, we initially examine the structure of $H(\gamma)$. To this end, we observe from (3.21) and (6.23) that all the antifields

$$\pi^{*\Theta} \equiv \left(\Pi^{*\Delta}, h^{*\mu\nu}, \eta^{*\mu}\right), \tag{6.40}$$

(where $\Pi^{*\Delta}$ are given in (5.10)) and their derivatives are non-trivial elements from $H(\gamma)$. Meanwhile, as γ acts on the original tensor fields via the gauge transformations (2.8) and (6.3) with the gauge parameters replaced by the ghosts, it results that the objects pertaining to $H(\gamma)$ constructed out of these fields coincide with the gauge-invariant ones, so they are the curvature tensor (2.29), the linearized Riemann tensor (6.8), as well as all their derivatives. (They are γ-non-trivial.) From (6.22) we observe that the undifferentiated Pauli-Fierz ghost η_μ and its antisymmetric first-order derivatives $\partial_{[\mu}\eta_{\nu]}$ are non-trivial elements pertaining to $H(\gamma)$, while the symmetric part of its first-order derivatives is γ-exact (see the former relation in (6.22)), and so are all their second- and higher-order derivatives since

$$\partial_\alpha\partial_\beta\eta_\mu = \frac{1}{2}\gamma\left(\partial_{(\alpha}h_{\beta)\mu} - \partial_\mu h_{\alpha\beta}\right). \tag{6.41}$$

[6]We stress again that the assumption on $a^{\mathrm{h-t}}$ ending at a finite antighost number is not a restriction; it only ensures that the corresponding interacting lagrangian $a_0^{\mathrm{h-t}}$ contains a finite, but otherwise arbitrary, number of derivatives.

[7]The fact that it is possible to replace the equation $\gamma a_I^{\mathrm{h-t}} = \partial_\mu \overset{(I)^\mu}{w}$ with (6.37) can be done like in the proof of corollary 1 from appendix A.

Related to the theory (2.1), we have determined in section 5 the non-trivial objects from $H(\gamma)$ involving only the ghosts and their derivatives. They are contained in the array (5.25), to which we further address. Thus, the general local solution to (6.37) is expressed (up to trivial, γ-exact contributions) by

$$a_I^{\mathrm{h-t}} = \alpha_I^{\mathrm{h-t}}\left(\left[\pi^{*\Theta}\right], \left[F_{\lambda\mu\nu\xi|\alpha\beta}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right) e^I\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, C_\nu, \mathcal{F}_{\lambda\mu\nu\alpha}\right), \tag{6.42}$$

for $I > 0$, where the γ-invariant objects $\alpha_I^{\mathrm{h-t}}$ (of pure ghost number equal to zero) are required to fulfill $\mathrm{agh}\left(\alpha_I^{\mathrm{h-t}}\right) = I$, and the notation e^I stands for a generic notation of the elements with pure ghost number equal to I of a basis in the space of polynomials in the corresponding ghosts and antisymmetrized first-order derivatives (this space is again finite-dimensional, like in the case of the theory (2.1) alone, since all the ghosts anticommute). In addition, every single term in $a_I^{\mathrm{h-t}}$ must contain at least one generator (field or ghost or antifield) from each of the two theories in order to provide effective cross-interactions. As they have a bounded number of derivatives, the quantities $\alpha_I^{\mathrm{h-t}}$ are in fact polynomials in the antifields, in the curvature tensor (2.29), in the linearized Riemann tensor, and also in their derivatives. They represent the most general non-trivial elements from $H^0(\gamma)$ and will again be called "invariant polynomials" (for the larger free gauge theory (6.1), subject to the gauge symmetries (2.8) and (6.3)).

Substituting the solution (6.42) into the equation (6.38) and taking into account the definitions (3.17)–(3.21) and (6.22– 6.23), we obtain, like in the previous investigated situation, that a necessary condition for the equation (6.38) to possess (non-trivial) solutions with respect to $a_{I-1}^{\mathrm{h-t}}$ for all $I > 0$ is that the polynomials $\alpha_I^{\mathrm{h-t}}$ are (non-trivial) objects from the local cohomology group of the Koszul-Tate differential at antighost number I and pure ghost number zero, $H_I(\delta|d)$

$$\delta\alpha_I^{\mathrm{h-t}} = \partial_\mu \overset{(I-1)^\mu}{k}, \qquad \mathrm{agh}\Big(\overset{(I-1)^\mu}{k}\Big) = I-1 \geq 0, \qquad \mathrm{pgh}\Big(\overset{(I-1)^\mu}{k}\Big) = 0. \tag{6.43}$$

Since the present model is a linear gauge theory of Cauchy order equal to four, we have that [26, 28]

$$H_I(\delta|d) = 0. \tag{6.44}$$

Meantime, it remains valid the result that if the invariant polynomial $\alpha_I^{\mathrm{h-t}}$ is trivial in $H_I(\delta|d)$ for $I \geq 4$, then it can be chosen to be trivial also in[8] $H_I^{\mathrm{inv}}(\delta|d)$

$$\left(\alpha_I^{\mathrm{h-t}} = \delta b_{I+1} + \partial_\mu \overset{(I)^\mu}{c}, \ \mathrm{agh}\left(\alpha_I^{\mathrm{h-t}}\right) = I \geq 4\right) \quad \Rightarrow \quad \alpha_I^{\mathrm{h-t}} = \delta\beta_{I+1} + \partial_\mu \overset{(I)^\mu}{\gamma}, \tag{6.45}$$

with both β_{I+1} and $\overset{(I)^\mu}{\gamma}$ invariant polynomials. As a consequence, we have that

$$H_I^{\mathrm{inv}}(\delta|d) = 0, \qquad I > 4, \tag{6.46}$$

where $H_I^{\mathrm{inv}}(\delta|d)$ denotes, like before, the local cohomology group of the Koszul-Tate differential at antighost number I in the space of invariant polynomials. On account of

[8] The proof can be realized in the same manner like theorem 3 from appendix B, with the precaution to include in an appropriate manner the dependence on the Pauli-Fierz sector.

the definitions (3.29) and (6.25), we are able to identify the non-trivial representatives of $\left(H_I\left(\delta|d\right)\right)_{I=2,3,4}$, as well as of $\left(H_I^{\mathrm{inv}}\left(\delta|d\right)\right)_{I=2,3,4}$, under the form

agh	non-trivial representatives spanning $H_I\left(\delta|d\right)$ and $H_I^{\mathrm{inv}}\left(\delta|d\right)$
$I>4$	none
$I=4$	$C^{*\nu}$
$I=3$	$\mathcal{C}'^{*\nu\alpha}$
$I=2$	$\mathcal{G}'^{*\mu\nu\vert\alpha}, \eta^{*\mu}$

(6.47)

Similar to the case of the free theory describing the mixed symmetry-type tensor field $t_{\lambda\mu\nu|\alpha}$, we note that here we also have no non-trivial element in $\left(H_I\left(\delta|d\right)\right)_{I\geq 2}$ or $\left(H_I^{\mathrm{inv}}\left(\delta|d\right)\right)_{I\geq 2}$ that involves the curvature tensors and/or their derivatives, the same observation being valid with respect to the objects that are more than linear in the antifields and/or depend on their derivatives. Like before, we require that $a^{\mathrm{h-t}}$ is Poincaré-invariant, so we exclude from its last representative, (6.42), the presence of non-trivial invariant polynomials α_I belonging to $H_I\left(\delta|d\right)$ or $H_I^{\mathrm{inv}}\left(\delta|d\right)$ that effectively depend on the space-time co-ordinates. As for the cohomological group $H_1\left(\delta|d\right)$, its determination is a difficult task, but we will solve the deformation equations without explicitly computing it.

The cohomology groups $H_I\left(\delta|d\right)$ and $H^{\mathrm{inv}}\left(\delta|d\right)$ at strictly positive antighost numbers give us information on the obstructions to remove the antifields from the first-order deformation. As a consequence of the result (6.46), we can eliminate all the terms with $I>4$ from the expansion (6.36) by adding to it only trivial pieces, and thus work with $I\leq 4$. This can be done like in appendix C. The last representative of (6.36) is of the type (6.42), with the corresponding invariant polynomials necessarily non-trivial in $H_I^{\mathrm{inv}}\left(\delta|d\right)$ for $I=2,3,4$, respectively in $H_1\left(\delta|d\right)$ for $I=1$.

6.3 The case $I=4$

For $I=4$, the first-order deformation (6.36) becomes

$$a^{\mathrm{h-t}}=a_0^{\mathrm{h-t}}+a_1^{\mathrm{h-t}}+a_2^{\mathrm{h-t}}+a_3^{\mathrm{h-t}}+a_4^{\mathrm{h-t}}\,, \tag{6.48}$$

with the γ-non-trivial part of $a_4^{\mathrm{h-t}}$ of the form (6.42)

$$a_4^{\mathrm{h-t}}=\alpha_4^{\mathrm{h-t}}e^4\left(\eta_\mu,\partial_{[\mu}\eta_{\nu]},C_\nu,\mathcal{F}_{\lambda\mu\nu\alpha}\right)+\gamma b_4\,, \tag{6.49}$$

and the invariant polynomial $\alpha_4^{\mathrm{h-t}}$ a non-trivial object from $H_4^{\mathrm{inv}}\left(\delta|d\right)$. We maintain the requirement on the maximum derivative order of $a_0^{\mathrm{h-t}}$ being equal to two and observe that, according to (6.47), $\alpha_4^{\mathrm{h-t}}$ can only be linear in the undifferentiated antifield $C^{*\nu}$. Consequently, we must select from the elements e^4 with pure ghost number equal to four only those having at most one space-time derivative (the terms with two or more derivatives lead, according to the relations (3.29), (3.32), (5.18) and (6.22), to corresponding pieces in $a_0^{\mathrm{h-t}}$ containing at least three derivatives). Meanwhile, we must be careful to include at least one ghost field from the Pauli-Fierz sector (otherwise, we go back to the problem of constructing consistent self-interactions for the tensor field $t_{\lambda\mu\nu|\alpha}$, which we have seen

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in the previous section that exhibits no consistent non-trivial solutions). Therefore, we remain only with the following elements e^4 obeying all these conditions

$$\text{eligible } e^4 : \left(\eta_\mu\eta_\nu\eta_\rho\eta_\sigma, \eta_\mu\eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]}, \eta_\mu\eta_\nu\eta_\rho\mathcal{F}_{\lambda'\mu'\nu'\rho'}, C_\mu\eta_\nu, C_\mu\partial_{[\nu}\eta_{\rho]}\right), \tag{6.50}$$

which then yield

$$\begin{aligned} a_4^{\mathrm{h-t}} = C_\lambda^* \Big(\eta_\mu\eta_\nu\eta_\rho \Big(f_1^{\lambda\mu\nu\rho\sigma}\eta_\sigma + f_2^{\lambda\mu\nu\rho\sigma\tau}\partial_{[\sigma}\eta_{\tau]} + f_3^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}\mathcal{F}_{\lambda'\mu'\nu'\rho'} \Big) + \\ + C_\mu \Big(f_4^{\lambda\mu\nu}\eta_\nu + f_5^{\lambda\mu\nu\rho}\partial_{[\nu}\eta_{\rho]} \Big) \Big) + \gamma b_4 . \end{aligned} \tag{6.51}$$

All the coefficients denoted by f must be constant (neither derivative nor depending on the space-time co-ordinates). Recalling that we work in $D \geq 5$ space-time dimensions, we find the following admitted representatives:

- in $D = 5$ dimensions
$$a_4^{(D=5)\mathrm{h-t}} = c_1\varepsilon^{\lambda\mu\nu\rho\sigma}C_\lambda^*\eta_\mu\eta_\nu\eta_\rho\eta_\sigma + \gamma b_4^{(D=5)}; \tag{6.52}$$
- in $D = 6$ dimensions
$$a_4^{(D=6)\mathrm{h-t}} = c_2\varepsilon^{\lambda\mu\nu\rho\sigma\tau}C_\lambda^*\eta_\mu\eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]} + \gamma b_4^{(D=6)}; \tag{6.53}$$
- in $D = 8$ dimensions
$$a_4^{(D=8)\mathrm{h-t}} = c_3\varepsilon^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}C_\lambda^*\eta_\mu\eta_\nu\eta_\rho\mathcal{F}_{\lambda'\mu'\nu'\rho'} + \gamma b_4^{(D=8)}; \tag{6.54}$$
- in all $D \geq 5$ dimensions
$$a_4^{(D)\mathrm{h-t}} = C^{*\lambda}\Big(c_2'\eta_\lambda\eta^\sigma\eta^\tau\partial_{[\sigma}\eta_{\tau]} + c_3'\eta^\mu\eta^\nu\eta^\rho\mathcal{F}_{\lambda\mu\nu\rho} + c_5'C^\mu\partial_{[\lambda}\eta_{\mu]}\Big) + \gamma b_4^{(D)} . \tag{6.55}$$

Direct computation shows that the terms containing at least one undifferentiated Pauli-Fierz ghost cannot produce a consistent component of antighost number three in (6.48), irrespective of the γ-exact contribution from (6.52)–(6.55), and hence we must set

$$c_1 = c_2 = c_3 = c_2' = c_3' = 0 , \tag{6.56}$$

which leaves us with a single candidate, namely,

$$a_4^{\mathrm{h-t}} = c_5'C^{*\lambda}C^\mu\partial_{[\lambda}\eta_{\mu]} + \gamma b_4 . \tag{6.57}$$

We will now show that (6.57) is not consistent at antighost number two, i.e., the equation

$$\delta a_4^{\mathrm{h-t}} + \gamma a_3^{\mathrm{h-t}} = \partial_\mu \overset{(3)}{w}{}^\mu , \tag{6.58}$$

possesses non-trivial solutions with respect to $a_3^{\mathrm{h-t}}$, while the next equation

$$\delta a_3^{\mathrm{h-t}} + \gamma a_2^{\mathrm{h-t}} = \partial_\mu \overset{(2)}{w}{}^\mu , \tag{6.59}$$

exhibits no non-zero solutions for $a_2^{\mathrm{h-t}}$. It is convenient (in order to simplify later developments) to fix the γ-exact term in (6.57) to the value

$$b_4 = c_5' C^{*\lambda} C^{\mu} h_{\lambda\mu}\,, \tag{6.60}$$

which further gives, on behalf of the last definitions in (3.20)–(3.21) combined with the first relation from (6.22)

$$a_4^{\mathrm{h-t}} = 2c_5' C^{*\lambda} C^{\mu} \partial_{\lambda}\eta_{\mu}\,. \tag{6.61}$$

Inserting (6.61) in (6.58), straightforward calculations produce

$$a_3^{\mathrm{h-t}} = 2c_5' \mathcal{C}'^{*\lambda\mu} \left(\mathcal{C}'_{\lambda}{}^{\nu} \partial_{\mu}\eta_{\nu} + 3C^{\nu}\left(\partial_{(\lambda}h_{\mu)\nu} - \partial_{\nu}h_{\lambda\mu}\right)\right), \tag{6.62}$$

and thus we then get

$$\begin{aligned}\delta a_3^{\mathrm{h-t}} = {} & \partial_{\mu}\left(-4c_5' \mathcal{G}'^{*\mu\nu|\lambda}\left(\mathcal{C}'_{\nu}{}^{\rho}\partial_{\lambda}\eta_{\rho} + 3C^{\rho}\left(\partial_{(\lambda}h_{\nu)\rho} - \partial_{\rho}h_{\lambda\nu}\right)\right)\right) + \\ & +\gamma\left(-2c_5' \mathcal{G}'^{*\mu\nu|\lambda}\left(\mathcal{G}'_{\mu\nu|}{}^{\rho}\partial_{\lambda}\eta_{\rho} + \mathcal{C}'_{\mu}{}^{\rho}\left(\partial_{(\lambda}h_{\nu)\rho} - \partial_{\rho}h_{\lambda\nu}\right)\right)\right) - \\ & -12c_5' \mathcal{G}'^{*\mu\nu|\alpha} K_{\mu\nu|\alpha\beta} C^{\beta}\,,\end{aligned} \tag{6.63}$$

with $K_{\mu\nu|\alpha\beta}$ the linearized Riemann tensor (6.8) and the prime variables defined in (3.27) and (3.30). Comparing (6.59) with (6.63), we can state that (6.59) admits solutions for $a_2^{\mathrm{h-t}}$ if and only if

$$-12c_5' \mathcal{G}'^{*\mu\nu|\alpha} K_{\mu\nu|\alpha\beta} C^{\beta} = \gamma b_2 + \partial_{\mu} \overset{(2)}{m}{}^{\mu}\,, \tag{6.64}$$

for some b_2 and $\overset{(2)}{m}{}^{\mu}$ with the properties

$$\mathrm{agh}\,(b_2) = 2 = \mathrm{agh}\left(\overset{(2)}{m}{}^{\mu}\right), \qquad \mathrm{pgh}\,(b_2) = 2\,, \qquad \mathrm{pgh}\left(\overset{(2)}{m}{}^{\mu}\right) = 3\,. \tag{6.65}$$

We find ourselves in a situation similar to that from the end of the subsection 5.2. Indeed, we remark that the left-hand side of (6.64) is a non-trivial element from $H^3(\gamma)$ of antighost number two, with the accompanying invariant polynomial of the form $-12c_5' \mathcal{G}'^{*\mu\nu|\alpha} K_{\mu\nu|\alpha\beta}$, such that we must set $b_2 = 0$, while the same expression cannot be written like a divergence, so we also have that $\overset{(2)}{m}{}^{\mu} = 0$. The above observations lead to the conclusion that (6.64) holds if and only if

$$c_5' = 0\,, \tag{6.66}$$

which further implies, via (6.61),

$$a_4^{\mathrm{h-t}} = 0\,. \tag{6.67}$$

Consequently, the first-order deformation (6.36) that describes the cross-interactions between the Pauli-Fierz field and the mixed symmetry-type tensor $t_{\lambda\mu\nu|\alpha}$ can be taken to stop at an antighost number $I \leq 3$.

6.4 The case $I = 3$

Assuming now that $I = 3$, we have that the first-order deformation (6.36) reduces to

$$a^{\mathrm{h-t}} = a_0^{\mathrm{h-t}} + a_1^{\mathrm{h-t}} + a_2^{\mathrm{h-t}} + a_3^{\mathrm{h-t}}\,, \tag{6.68}$$

where the last representative is

$$a_3^{\mathrm{h-t}} = \alpha_3^{\mathrm{h-t}} e^3 \left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, C_\nu, \mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_3\,, \tag{6.69}$$

with $\alpha_3^{\mathrm{h-t}}$ non-trivial in $H_3^{\mathrm{inv}}(\delta|d)$. Following the same line like in the previous subsection, we apply the result from (6.47), according to which $\alpha_3^{\mathrm{h-t}}$ is linear in the antifield $\mathcal{C}'^{*}_{\lambda\mu}$, such that the elements e^3 that can be used to construct an $a_3^{\mathrm{h-t}}$ with the desired properties (fulfilling the derivative order assumption and providing effective cross-interactions between the two types of tensor fields) are spanned by

$$\text{eligible } e^3 : \left(\eta_\nu\eta_\rho\eta_\sigma, \eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]}, \eta_\nu\eta_\rho\mathcal{F}_{\lambda'\mu'\nu'\rho'}\right), \tag{6.70}$$

so we obtain that

$$a_3^{\mathrm{h-t}} = \mathcal{C}'^{*}_{\lambda\mu}\eta_\nu\eta_\rho \left(f_1^{\lambda\mu\nu\rho\sigma}\eta_\sigma + f_2^{\lambda\mu\nu\rho\sigma\tau}\partial_{[\sigma}\eta_{\tau]} + f_3^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}\mathcal{F}_{\lambda'\mu'\nu'\rho'}\right) + \gamma b_3. \tag{6.71}$$

All the coefficients denoted by f are restricted, as everywhere before, to be constant. By organizing the emerging acceptable (Lorentz-covariant and Poincaré-invariant) combinations according to the space-time dimension, we consequently arrive at:

- in $D = 5$ dimensions
$$a_3^{(D=5)\mathrm{h-t}} = c_1\varepsilon^{\lambda\mu\nu\rho\sigma}\mathcal{C}'^{*}_{\lambda\mu}\eta_\nu\eta_\rho\eta_\sigma + \gamma b_3^{(D=5)}\,; \tag{6.72}$$
- in $D = 6$ dimensions
$$a_3^{(D=6)\mathrm{h-t}} = c_2\varepsilon^{\lambda\mu\nu\rho\sigma\tau}\mathcal{C}'^{*}_{\lambda\mu}\eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]} + \gamma b_3^{(D=6)}\,; \tag{6.73}$$
- in $D = 8$ dimensions
$$a_3^{(D=8)\mathrm{h-t}} = c_3\varepsilon^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}\mathcal{C}'^{*}_{\lambda\mu}\eta_\nu\eta_\rho\mathcal{F}_{\lambda'\mu'\nu'\rho'} + \gamma b_3^{(D=8)}\,; \tag{6.74}$$
- in all $D \geq 5$ dimensions
$$a_3^{(D)\mathrm{h-t}} = \mathcal{C}'^{*\lambda\mu}\left(c_2'\sigma_{\lambda\mu}\eta^\nu\eta^\rho\partial_{[\nu}\eta_{\rho]} + c_2''\eta_\lambda\eta^\nu\partial_{[\mu}\eta_{\nu]} + c_3'\eta^\nu\eta^\rho\mathcal{F}_{\lambda\mu\nu\rho}\right) + \gamma b_3^{(D)}\,. \tag{6.75}$$

Straightforward computation shows that (6.72)–(6.75) cannot be lifted to antighost number two, i.e., there are no solutions $a_2^{\mathrm{h-t}}$ to the equation $\delta a_3^{\mathrm{h-t}} + \gamma a_2^{\mathrm{h-t}} = \partial_\mu \overset{(2)}{w}{}^{\mu}$, irrespective of what γ-exact contributions we take in their right-hand sides, so we must set

$$c_1 = c_2 = c_3 = c_2' = c_2'' = c_3' = 0, \tag{6.76}$$

which leads to

$$a_3^{\mathrm{h-t}} = 0\,, \tag{6.77}$$

and so the first-order deformation $a^{\mathrm{h-t}}$ cannot end at antighost number three either.

6.5 The case $I = 2$

The next possible maximum value of the antighost number in the expansion (6.36) is $I = 2$, in which case $a^{\mathrm{h-t}}$ reads

$$a^{\mathrm{h-t}} = a_0^{\mathrm{h-t}} + a_1^{\mathrm{h-t}} + a_2^{\mathrm{h-t}}\,, \tag{6.78}$$

where $a_2^{\mathrm{h-t}}$ belongs to $H^2(\gamma)$, and hence is of the form

$$a_2^{\mathrm{h-t}} = \alpha_2^{\mathrm{h-t}} e^2\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_2\,, \tag{6.79}$$

and $\alpha_2^{\mathrm{h-t}}$ is a non-trivial object from $H_2^{\mathrm{inv}}(\delta|d)$. We have seen in the subsection 6.2, formula (6.47), that $H_2^{\mathrm{inv}}(\delta|d)$ is spanned by the antifields $\mathcal{G}'^{*\mu\nu|\alpha}$ and $\eta^{*\mu}$. This is actually the first place where the Pauli-Fierz theory brings non-trivial contributions to the local cohomology of the Koszul-Tate differential. Taking into account the actions of δ on these antifields (the first definition in (3.29), respectively the last one in (6.25)), and also on those with antighost number one (the first relation in (3.23) and (6.25) respectively), we observe that e^2 cannot include more than one space-time derivative in order to ensure an $a_0^{\mathrm{h-t}}$ with at most two derivatives. Meanwhile, for $a^{\mathrm{h-t}}$ to describe cross-interactions, the terms proportional with $\mathcal{G}'^{*\mu\nu|\alpha}$ must involve at least one of the combinations η_μ or $\partial_{[\mu}\eta_{\nu]}$, while those linear in $\eta^{*\mu}$ are required to depend on $\mathcal{F}_{\lambda\mu\nu\alpha}$. The above considerations render the eligible e^2 like

$$\text{eligible } e^2:\ \left(\eta_\nu\eta_\rho, \eta_\nu\partial_{[\rho}\eta_{\sigma]}, \eta_\nu\mathcal{F}_{\lambda'\mu'\nu'\rho'}\right)\,, \tag{6.80}$$

which, appended to the general assumptions of Lorentz covariance and Poincaré invariance, then yield

$$\begin{aligned} a_2^{\mathrm{h-t}} &= \mathcal{G}'^{*}_{\lambda\mu|\alpha}\eta_\nu\left(f_1^{\lambda\mu\alpha\nu\rho}\eta_\rho + f_2^{\lambda\mu\alpha\nu\rho\sigma}\partial_{[\rho}\eta_{\sigma]} + f_3^{\lambda\mu\alpha\nu\lambda'\mu'\nu'\rho'}\mathcal{F}_{\lambda'\mu'\nu'\rho'}\right) + \\ &\quad + f_4^{\lambda\mu\lambda'\mu'\nu'\rho'}\eta^*_\lambda\eta_\mu\mathcal{F}_{\lambda'\mu'\nu'\rho'} + \gamma b_2\,, \end{aligned} \tag{6.81}$$

where all the f's are constant. Structuring the independent possibilities like in the above, according to the space-time dimension, we get:

- in $D = 5$ dimensions

$$a_2^{(D=5)\mathrm{h-t}} = c_1\varepsilon^{\lambda\mu\alpha\nu\rho}\mathcal{G}'^{*}_{\lambda\mu|\alpha}\eta_\nu\eta_\rho + \gamma b_2^{(D=5)}\,; \tag{6.82}$$

- in $D = 6$ dimensions

$$\begin{aligned} a_2^{(D=6)\mathrm{h-t}} &= c_2\varepsilon^{\lambda\mu\alpha\nu\rho\sigma}\mathcal{G}'^{*}_{\lambda\mu|\alpha}\eta_\nu\partial_{[\rho}\eta_{\sigma]} + \\ &\quad + c_4\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'}\eta^*_\lambda\eta_\mu\mathcal{F}_{\lambda'\mu'\nu'\rho'} + \gamma b_2^{(D=6)}\,; \end{aligned} \tag{6.83}$$

- in $D = 8$ dimensions

$$a_2^{(D=8)\mathrm{h-t}} = c_3\varepsilon^{\lambda\mu\alpha\nu\lambda'\mu'\nu'\rho'}\mathcal{G}'^{*}_{\lambda\mu|\alpha}\eta_\nu\mathcal{F}_{\lambda'\mu'\nu'\rho'} + \gamma b_2^{(D=8)}\,; \tag{6.84}$$

- in all $D \geq 5$ dimensions

$$a_2^{(D)\mathrm{h-t}} = \mathcal{G}'^{*\lambda\mu|\alpha}\left(c'_2\eta_\alpha\partial_{[\lambda}\eta_{\mu]} + c''_2\sigma_{\mu\alpha}\eta^\mu\partial_{[\lambda}\eta_{\mu]} + c'_3\eta^\nu\mathcal{F}_{\lambda\mu\alpha\nu}\right) + \gamma b_2^{(D)}\,. \tag{6.85}$$

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Due to the presence in all the previous representatives of $H^2(\gamma)$ of at least one undifferentiated Pauli-Fierz ghost, it results that they cannot be appropriately lifted to $a_1^{\mathrm{h-t}}$ as solution to the equation $\delta a_2^{\mathrm{h-t}} + \gamma a_1^{\mathrm{h-t}} = \partial_\mu \overset{(1)^\mu}{w}$, such that we are obliged to take

$$c_1 = c_2 = c_4 = c_3 = c_2' = c_2'' = c_3' = 0\,, \tag{6.86}$$

which further produces

$$a_2^{\mathrm{h-t}} = 0\,, \tag{6.87}$$

so we conclude that the first-order deformation $a^{\mathrm{h-t}}$ can only stop at $I \leq 1$. Thus, there are no non-trivial cross-interactions between the tensor field $t_{\lambda\mu\nu|\alpha}$ and the Pauli-Fierz field complying with all the above mentioned requirements that modify the original abelian gauge algebra (type (iii) cross-couplings).

6.6 The case $I = 1$

For $I = 1$, the first-order deformation (6.36) reduces to

$$a^{\mathrm{h-t}} = a_0^{\mathrm{h-t}} + a_1^{\mathrm{h-t}}\,, \tag{6.88}$$

with $a_1^{\mathrm{h-t}}$ from $H^1(\gamma)$

$$a_1^{\mathrm{h-t}} = \alpha_1^{\mathrm{h-t}} e^1\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_1\,, \tag{6.89}$$

and $\alpha_1^{\mathrm{h-t}}$ necessarily an element from $H_1(\delta|d)$, such that

$$\alpha_1^{\mathrm{h-t}} = \alpha_1^{\mathrm{h-t}}\left(\left[t^{*\lambda\mu\nu|\alpha}\right], \left[h^{*\mu\nu}\right], \left[F_{\lambda\mu\nu\xi|\alpha\beta}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right)\,, \qquad \delta\alpha_1^{\mathrm{h-t}} = \partial_\mu t^\mu\,. \tag{6.90}$$

The elements of pure ghost number equal to one of a basis in the space of polynomials in η_μ, $\partial_{[\mu}\eta_{\nu]}$ and $\mathcal{F}_{\lambda\mu\nu\alpha}$ are spanned by

$$e^1 : \left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}_{\lambda\mu\nu\alpha}\right)\,, \tag{6.91}$$

while the invariant polynomial $\alpha_1^{\mathrm{h-t}} \in H_1(\delta|d)$ is linear in the antifields $t^{*\lambda\mu\nu|\alpha}$, $h^{*\mu\nu}$ and in their derivatives up to some finite orders, as these are the only generators of antighost number equal to one from the BRST complex. The group $H_1(\delta|d)$ at pure ghost number zero is infinite-dimensional. However, the assumption on the maximum derivative order of $a_0^{\mathrm{h-t}}$ being equal to two helps us in avoiding the difficult task of computing $H_1(\delta|d)$. More precisely, this requirement restricts $\alpha_1^{\mathrm{h-t}}$ not to depend on either the curvatures or their derivatives. Indeed, assuming that $\alpha_1^{\mathrm{h-t}}$ involves any of the curvatures, then the component from (6.90) with the minimum number of derivatives is linear in the undifferentiated antifields, in the undifferentiated curvatures, as well as in the undifferentiated Pauli-Fierz ghost. If consistent, it would produce an $a_0^{\mathrm{h-t}}$ with three space-time derivatives. Along the same line, $\alpha_1^{\mathrm{h-t}}$ cannot contain more that the first-order derivatives of the antifields. Regarding the terms including the first-order derivatives of the antifields, we can always make (by an integration by parts in the corresponding functional) the derivative to act on e^1, and therefore they can be taken to be linear in the undifferentiated antifields. As the first-order derivatives of $\partial_{[\mu}\eta_{\nu]}$ and $\mathcal{F}_{\lambda\mu\nu\alpha}$ are γ-exact, the corresponding terms in $a_1^{\mathrm{h-t}}$ can be discarded since they are γ-trivial. The piece displaying the first-order derivatives of η_μ can always be made proportional with $\partial_{[\mu}\eta_{\nu]}$ by adding irrelevant, γ-exact objects, so it can be taken to depend only on the undifferentiated antifields. In consequence, the dependence of $\alpha_1^{\mathrm{h-t}}$ on the first-order derivatives of the antifields can be removed. Accordingly, we can write that

$$a_1^{\mathrm{h-t}} = \alpha_{1\mathrm{lin}}^{\mathrm{h-t}}\left(t^{*\lambda\mu\nu|\alpha}, h^{*\mu\nu}\right) e^1\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}_{\lambda\mu\nu\alpha}\right) + \gamma b_1 \,. \tag{6.92}$$

Selecting only the non-trivial terms that potentially lead to cross-couplings among the two types of tensor fields, we arrive at

$$a_1^{\mathrm{h-t}} = t^*_{\lambda\mu\nu|\alpha}\left(f_1^{\lambda\mu\nu\alpha\rho}\eta_\rho + f_2^{\lambda\mu\nu\alpha\rho\sigma}\partial_{[\rho}\eta_{\sigma]}\right) + f_3^{\lambda\mu\nu\rho\sigma\alpha}h^*_{\lambda\mu}\mathcal{F}_{\nu\rho\sigma\alpha} + \gamma b_1 \,, \tag{6.93}$$

where all the coefficients denoted by f are required to be constant. Taking into account the identity (3.3) satisfied by the antifield $t^*_{\lambda\mu\nu|\alpha}$, it follows that any solution containing Levi-Civita symbols contracted on all the indices of this antifield vanish. Invoking in addition the symmetry of the Pauli-Fierz antifield, we remain with a single candidate in all $D \geq 5$ dimensions

$$a_1^{\mathrm{h-t}} = c t^{*\lambda\mu}\partial_{[\lambda}\eta_{\mu]} \,, \tag{6.94}$$

where the notation $t^{*\lambda\mu}$ is explained in (3.6). (For convenience, we took $b_1 = 0$ in (6.93).) We observe that the first definition from (3.23) together with the identity (2.27) yield $\delta a_1^{\mathrm{h-t}}$ under a divergence-like form

$$\delta a_1^{\mathrm{h-t}} = \partial_\mu\left(2c\eta_\lambda T^{\lambda\mu}\right), \tag{6.95}$$

where $T^{\lambda\mu}$ is expressed in (2.23). Consequently, the corresponding $a_0^{\mathrm{h-t}}$ as solution to the equation

$$\delta a_1^{\mathrm{h-t}} + \gamma a_0^{\mathrm{h-t}} = \partial_\mu \overset{(0)}{w}{}^{\mu} \,, \tag{6.96}$$

can be taken equal to zero

$$a_0^{\mathrm{h-t}} = 0 \,, \tag{6.97}$$

so we obtain that the first-order deformation (6.88) reduces to its antighost number one component

$$a^{\mathrm{h-t}} = a_1^{\mathrm{h-t}} = c t^{*\lambda\mu}\partial_{[\lambda}\eta_{\mu]} \,. \tag{6.98}$$

This solution presents a strange behavior. It modifies the gauge transformations of the tensor field $t_{\lambda\mu\nu|\alpha}$ at order one in the coupling constant by elements involving the Pauli-Fierz gauge parameter, but adds no coupling terms with the Pauli-Fierz field to the deformed lagrangian action. Actually, the solution (6.98) is purely trivial in $H^0(s|d)$. In order to see this, we observe that (6.95) is nothing but the standard equation satisfied by the elements from the local cohomology of the Koszul-Tate differential, and therefore $a_1^{\mathrm{h-t}}$ belongs to $H(\delta|d)$ at antighost number one *and* pure ghost number one. According to the footnote 4, this cohomology is trivial, i.e., $a_1^{\mathrm{h-t}} = \delta c_2^{\mathrm{h-t}} + \partial_\mu j_1^\mu$, for some $c_2^{\mathrm{h-t}}$ and j_1^μ with agh $\left(c_2^{\text{h-t}}\right) = 2$, agh $\left(j_1^\mu\right) = $ pgh $\left(c_2^{\text{h-t}}\right) = $ pgh $\left(j_1^\mu\right) = 1$. Indeed, from the first definition in (3.29) and the second relation in (6.24), we get that

$$a_1^{\text{h-t}} = \delta\left(-\frac{2c}{3}\mathcal{G}'^{*\mu\nu|\alpha}\sigma_{\nu\alpha}\eta_\mu\right) + \partial_\mu\left(-2ct^{*\lambda\mu}\eta_\lambda\right). \tag{6.99}$$

Due to the γ-invariance of the Pauli-Fierz ghost and of all the antifields, the relations (6.98)–(6.99) allow us to write

$$a^{\text{h-t}} = s\left(-\frac{2c}{3}\mathcal{G}'^{*\mu\nu|\alpha}\sigma_{\nu\alpha}\eta_\mu\right) + \partial_\mu\left(-2ct^{*\lambda\mu}\eta_\lambda\right), \tag{6.100}$$

and hence (6.98) is indeed trivial in $H^0(s|d)$, such that it can be safely removed from the first-order deformation. In conclusion, the deformation procedure allows no non-trivial cross-couplings that change the original gauge transformations (2.8) and (6.3) (type (ii) interactions).

6.7 The case $I = 0$

We are now left with one more case, where the first-order deformation (6.36) coincides with its antighost number zero component ($I = 0$)

$$a^{\text{h-t}} = a_0^{\text{h-t}}\left(\left[t_{\lambda\mu\nu|\alpha}\right], \left[h_{\mu\nu}\right]\right), \tag{6.101}$$

with

$$\gamma a_0^{\text{h-t}} = \partial_\mu \overset{(0)}{w}{}^\mu, \tag{6.102}$$

being understood that we discard the divergence-like solutions. As we have discussed in the subsection 5.5, there appear two different situations. The first one is associated with $\overset{(0)}{w}{}^\mu = 0$ in (6.102) and its solutions are constructed from the gauge-invariant quantities, which are the curvatures and their derivatives

$$\gamma a_0'^{\text{h-t}} = 0 \quad \Rightarrow \quad a_0'^{\text{h-t}} = a_0'^{\text{h-t}}\left(\left[F_{\lambda\mu\nu\xi|\alpha\beta}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right). \tag{6.103}$$

However, the cross-coupling component with the minimum number of derivatives from $a_0'^{\text{h-t}}$ is of order four in the space-time derivatives, being proportional with $F_{\lambda\mu\nu\xi|\alpha\beta}K_{\mu'\nu'|\alpha'\beta'}$, and therefore the solutions (6.103) are not eligible, as they disagree with the assumption on the maximum derivative order of the interacting lagrangian being equal to two.

The second situation corresponds to $\overset{(0)}{w}{}^\mu \neq 0$ in (6.102). Denoting the Euler-Lagrange derivatives of $a_0^{\text{h-t}}$ by

$$B^{\lambda\mu\nu|\alpha} \equiv \frac{\delta a_0^{\text{h-t}}}{\delta t_{\lambda\mu\nu|\alpha}}, \qquad D^{\mu\nu} \equiv \frac{\delta a_0^{\text{h-t}}}{\delta h_{\mu\nu}}, \tag{6.104}$$

and using the definition (3.17) together with the first relation from (6.22), the equation (6.102) further implies that

$$\partial_\alpha B^{\lambda\mu\nu|\alpha} = 0, \qquad \partial_\lambda B^{\lambda\mu\nu|\alpha} = 0, \qquad \partial_\mu D^{\mu\nu} = 0. \tag{6.105}$$

The tensor $B^{\lambda\mu\nu|\alpha}$ has the same mixed symmetry like $t_{\lambda\mu\nu|\alpha}$ and $D^{\mu\nu}$ is symmetric. Moreover, $B^{\lambda\mu\nu|\alpha}$ and $D^{\mu\nu}$ must involve at least one Pauli-Fierz field, respectively, one tensor field $t_{\lambda\mu\nu|\alpha}$ in order to provide cross-couplings. According to the considerations from the section 2 and the subsection 6.1 (see the formulas (2.45)–(2.46) and (6.18)–(6.19)), the solutions to the equations (6.105) are of the type[9]

$$\frac{\delta a_0^{\mathrm{h-t}}}{\delta t_{\lambda\mu\nu|\alpha}} \equiv B^{\lambda\mu\nu|\alpha} = \partial_\xi \partial_\beta \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}\,, \qquad \frac{\delta a_0^{\mathrm{h-t}}}{\delta h_{\mu\nu}} \equiv D^{\mu\nu} = \partial_\alpha \partial_\beta \bar{\Phi}^{\mu\alpha|\nu\beta}\,, \tag{6.106}$$

where $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$ depend only on the undifferentiated fields $h_{\mu\nu}$ and $t_{\lambda\mu\nu|\alpha}$ (otherwise, the corresponding $a_0^{\mathrm{h-t}}$ would be more than second-order in the derivatives), with $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ having the mixed symmetry of the curvature tensor $F^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$ that of the linearized Riemann tensor. From now on we proceed along the line employed in the subsection 5.6. In view of this, we introduce a derivation on the algebra of non-integrated densities depending on $t_{\lambda\mu\nu|\alpha}$, $h_{\mu\nu}$ and on their derivatives, that counts the powers of the fields and their derivatives

$$\begin{aligned}\bar{N} = \sum_{n\geq 0} \Bigg(&\left(\partial_{\mu_1}\cdots\partial_{\mu_n} t_{\lambda\mu\nu|\alpha}\right) \frac{\partial}{\partial\left(\partial_{\mu_1}\cdots\partial_{\mu_n} t_{\lambda\mu\nu|\alpha}\right)} + \\ &+ \left(\partial_{\mu_1}\cdots\partial_{\mu_n} h_{\mu\nu}\right) \frac{\partial}{\partial\left(\partial_{\mu_1}\cdots\partial_{\mu_n} h_{\mu\nu}\right)} \Bigg)\,,\end{aligned} \tag{6.107}$$

and observe that the action of $\bar{N}$ on an arbitrary non-integrated density $\bar{u}$ is

$$\bar{N}\bar{u} = t_{\lambda\mu\nu|\alpha} \frac{\delta\bar{u}}{\delta t_{\lambda\mu\nu|\alpha}} + h_{\mu\nu} \frac{\delta\bar{u}}{\delta h_{\mu\nu}} + \partial_\mu r^\mu\,, \tag{6.108}$$

where $\delta\bar{u}/\delta t_{\lambda\mu\nu|\alpha}$ and $\delta\bar{u}/\delta h_{\mu\nu}$ denote the variational derivatives of $\bar{u}$. In the case where $\bar{u}$ is an homogeneous polynomial of order $p > 0$ in the fields and their derivatives, we have that $\bar{N}\bar{u} = p\bar{u}$, and so

$$\bar{u} = \frac{1}{p}\left(t_{\lambda\mu\nu|\alpha} \frac{\delta\bar{u}}{\delta t_{\lambda\mu\nu|\alpha}} + h_{\mu\nu} \frac{\delta\bar{u}}{\delta h_{\mu\nu}} \right) + \partial_\mu \left(\frac{1}{p} r^\mu\right). \tag{6.109}$$

As a_0 can always be decomposed as a sum of homogeneous polynomials of various orders, it is enough to analyze the equation (6.102) for a fixed value of p. Putting $\bar{u} = a_0^{\mathrm{h-t}}$ in (6.109) and inserting (6.106) in the associated relation, we can write

$$a_0^{\mathrm{h-t}} = \frac{1}{p}\left(t_{\lambda\mu\nu|\alpha} \partial_\xi \partial_\beta \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta} + h_{\mu\nu} \partial_\alpha \partial_\beta \bar{\Phi}^{\mu\alpha|\nu\beta} \right) + \partial_\mu \bar{r}^\mu\,. \tag{6.110}$$

Integrating two times by parts in (6.110) and recalling the mixed symmetries of $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$, we infer that

$$a_0^{\mathrm{h-t}} = k_1 F_{\lambda\mu\nu\xi|\alpha\beta} \tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta} + k_2 K_{\mu\alpha|\nu\beta} \bar{\Phi}^{\mu\alpha|\nu\beta} + \partial_\mu \bar{l}^\mu\,, \tag{6.111}$$

[9]The solution $D^{\mu\nu} \sim \sigma^{\mu\nu}$ leads to $a_0 \sim h$ and it does not describe cross-couplings, but only a cosmological term for the spin-two field. This term is separately included among the consistent deformations of the Pauli-Fierz theory (see the formula (6.119)).

with $k_1 = 1/8p$ and $k_2 = -1/2p$. By computing the action of γ on (6.111) and following a reasoning similar to that applied between the formulas (5.86) and (5.93), we obtain that $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$ are precisely of the type

$$\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta} = k'\Phi^{\lambda\mu\nu\xi|\alpha\beta}, \qquad \bar{\Phi}^{\mu\alpha|\nu\beta} = k''\Phi^{\mu\alpha|\nu\beta}, \tag{6.112}$$

where k' and k'' are real constants, while $\Phi^{\lambda\mu\nu\xi|\alpha\beta}$ and $\Phi^{\mu\alpha|\nu\beta}$ are the tensors involved in the functions (2.47) and (6.20) that define free field equations, being respectively expressed by (2.48) and (6.21). Accordingly, the admitted value of p is fixed to $p = 2$. Replacing (6.112) in (6.110) for $p = 2$, we finally arrive at

$$a_0^{\mathrm{h-t}}\left(\left[t_{\lambda\mu\nu|\alpha}\right], \left[h_{\mu\nu}\right]\right) = \frac{k'}{2} t_{\lambda\mu\nu|\alpha} T^{\lambda\mu\nu|\alpha} + \frac{k''}{2} h_{\mu\nu} H^{\mu\nu} + \partial_\mu \bar{r}^\mu . \tag{6.113}$$

The solution (6.113) is not eligible since it gives no cross-couplings between the two types of investigated tensor fields. In fact, it is (up to an irrelevant divergence) a linear combination of the original free lagrangians, so it is trivial in $H^0(s|d)$. Therefore, we can always discard it from the first-order deformation

$$a_0^{\mathrm{h-t}}\left(\left[t_{\lambda\mu\nu|\alpha}\right], \left[h_{\mu\nu}\right]\right) = 0 . \tag{6.114}$$

In consequence, there are no cross-couplings invariant under the gauge transformations (2.8) and (6.3) that can be added to the free action (6.1) (type (i) interactions).

Putting together the results contained in this section, we can state that

$$S_1^{\mathrm{h-t}} = 0 , \tag{6.115}$$

and so

$$S_1 = S_1^{\mathrm{h-h}} , \tag{6.116}$$

where $S_1^{\mathrm{h-h}}$ is the first-order deformation of the solution to the master equation for the Pauli-Fierz theory. The consistency of the deformed solution to the master equation at the second order in the coupling constant is governed by the equation (4.5), where $(S_1^{\mathrm{h-h}}, S_1^{\mathrm{h-t}}) = 0 = (S_1^{\mathrm{h-t}}, S_1^{\mathrm{h-t}})$, but $(S_1^{\mathrm{h-h}}, S_1^{\mathrm{h-h}}) \neq 0$, and thus we have that $S_2^{\mathrm{h-t}} = 0$, while $S_2^{\mathrm{h-h}}$ is highly non-trivial and is known to describe the quartic vertex of the Einstein-Hilbert action, as well as the second-order contributions to the gauge transformations and to the associated non-abelian gauge algebra. The vanishing of $S_1^{\mathrm{h-t}}$ and $S_2^{\mathrm{h-t}}$ further leads, via the equations that stipulate the higher-order deformation equations, to the result that

$$S_k^{\mathrm{h-t}} = 0 , \qquad k \geq 1 . \tag{6.117}$$

The conclusion of this section is that, under the simultaneous requirements of Lorentz covariance, Poincaré invariance and space-time locality of the deformations, combined with the requirement on the maximum derivative order of the interacting lagrangian being equal to two, there are no non-trivial and consistent cross-interactions between the Pauli-Fierz field and the mixed symmetry-type tensor field $t_{\lambda\mu\nu|\alpha}$ in $D \geq 5$ dimensions.

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The only pieces that can be added to the action (6.1) are given by the self-interactions of the Pauli-Fierz field, so we can write that

$$S_0^{\rm int}\left[t_{\lambda\mu\nu|\alpha}, h_{\mu\nu}\right] = S^{\rm EH}\left[g_{\mu\nu}\right] + S_0\left[t_{\lambda\mu\nu|\alpha}\right], \tag{6.118}$$

where $S^{\rm EH}\left[g_{\mu\nu}\right]$ is the Einstein-Hilbert action

$$S^{\rm EH}\left[g_{\mu\nu}\right] = \frac{2}{\kappa^2}\int d^Dx\sqrt{-g}\left(R - 2\Lambda\right), \tag{6.119}$$

$$g_{\mu\nu} = \sigma_{\mu\nu} + \kappa h_{\mu\nu}, \qquad \kappa = g\overset{(1)}{\kappa} + g^2\overset{(2)}{\kappa} + \cdots . \tag{6.120}$$

Action (6.119) is known to be invariant under the deformed gauge transformations (diffeomorphisms)

$$\frac{1}{\kappa}\delta_\epsilon g_{\mu\nu} = \epsilon_{\mu;\nu} + \epsilon_{\nu;\mu}, \tag{6.121}$$

with $\epsilon_{\mu;\nu}$ the covariant derivative of ϵ_μ from General Relativity. In the above R represents the full scalar curvature for the metric $g_{\mu\nu}$, and Λ denotes the cosmological constant

$$\Lambda = g\overset{(1)}{\Lambda} + g^2\overset{(2)}{\Lambda} + \cdots . \tag{6.122}$$

7. Couplings to matter: broken Poincaré invariance

7.1 "Free" model and BRST description

The final scope of our paper is to analyze the possible consistent interactions between the tensor field $t_{\lambda\mu\nu|\alpha}$ and matter fields. In view of this, let us consider the "free" action

$$S_0\left[t_{\lambda\mu\nu|\alpha}, y^i\right] = S_0\left[t_{\lambda\mu\nu|\alpha}\right] + S^{\rm matt}\left[y^i\right], \tag{7.1}$$

where $S^{\rm matt}$ describes a generic matter theory

$$S^{\rm matt}\left[y^i\right] = \int d^Dx\mathcal{L}\left(\left[y^i\right]\right), \tag{7.2}$$

which is at most second-order derivative. The fields y^i are assumed to have no non-trivial gauge symmetries, the only "true" gauge transformations of the action (7.1) being those of the field $t_{\lambda\mu\nu|\alpha}$

$$\delta_{\epsilon,\chi}t_{\lambda\mu\nu|\alpha} = 3\partial_\alpha\epsilon_{\lambda\mu\nu} + \partial_{[\lambda}\epsilon_{\mu\nu]\alpha} + \partial_{[\lambda}\chi_{\mu\nu]|\alpha}, \qquad \delta_{\epsilon,\chi}y^i = 0. \tag{7.3}$$

In particular, the field equations of (7.1) with respect to $t_{\lambda\mu\nu|\alpha}$ are nothing but (2.20), with the corresponding functions fulfilling the Noether identities (2.26), while those for the matter fields

$$\frac{\delta^L S_0\left[t_{\lambda\mu\nu|\alpha}, y^i\right]}{\delta y^i} = \frac{\delta^L\mathcal{L}}{\delta y^i} \approx 0, \tag{7.4}$$

are expressed via independent left Euler-Lagrange derivatives $\delta^L\mathcal{L}/\delta y^i$. In the sequel we will denote the Grassmann parity of the field y^i by ε_i. We allow the matter action to include couplings among the fields y^i, but presume that the Cauchy order of this theory alone is equal to one. This is however not a restrictive assumption since all the usual matter theories satisfy it. Accordingly, we obtain that the Cauchy order of the entire theory (7.1) is equal to four. From (7.3) we observe that the most general gauge-invariant objects of this "free" model are polynomials in the curvature tensor $F_{\lambda\mu\nu\xi|\alpha\beta}$, in the matter fields y^i, as well as in all their derivatives.

The BRST differential of this system decomposes like in (3.7) and its components are still governed by the equations (3.16). The actions of γ and δ on the BRST generators corresponding to the tensor field $t_{\lambda\mu\nu|\alpha}$ are expressed by the formulas (3.17)–(3.26) (see also (3.29) and (3.32)), while on those from the matter sector read as

$$\gamma y^i = 0\,, \qquad \gamma y_i^* = 0\,, \qquad \delta y^i = 0\,, \qquad \delta y_i^* = -\frac{\delta^L \mathcal{L}}{\delta y^i}\,, \tag{7.5}$$

where

$$\mathrm{pgh}\left(y^i\right) = 0 = \mathrm{pgh}\left(y_i^*\right), \qquad \mathrm{agh}\left(y^i\right) = 0\,, \qquad \mathrm{agh}\left(y_i^*\right) = 1\,, \tag{7.6}$$

and y_i^* denote the antifields of the matter fields, of Grassmann parity $\varepsilon(y_i^*) = (\varepsilon_i + 1)\mathrm{mod}\ 2$. The canonical action of the BRST differential, $s\cdot = (\cdot, S)$, where the antibracket is extended such as to include the matter fields and their antifields, is generated by the solution to the master equation $(S, S) = 0$, which takes the simple form

$$S = S^{\mathrm{t}} + S^{\mathrm{matt}}\left[y^i\right], \tag{7.7}$$

where S^{t} is given by the right-hand side of (3.34).

7.2 $H(\gamma)$ and $H(\delta|d)$

The non-integrated density of the first-order deformation a for this model can be split like in the subsection 6.2, namely,

$$a = a^{\mathrm{m-m}} + a^{\mathrm{m-t}}\,, \tag{7.8}$$

where $a^{\mathrm{m-m}}$ governs the self-interactions of the matter fields and $a^{\mathrm{m-t}}$ controls the cross-interactions between y^i and $t_{\lambda\mu\nu|\alpha}$. (According to the general result of section 5, there is no non-trivial piece related to the self-interactions of the tensor field $t_{\lambda\mu\nu|\alpha}$.) Naturally, a is subject to the equation and conditions from the relation (5.1). Since the matter theory possesses no non-trivial gauge symmetries, it follows that $a^{\mathrm{m-m}}$ coincides with its antighost number zero component

$$a^{\mathrm{m-m}} = a_0^{\mathrm{m-m}}\left(\left[y^i\right]\right), \tag{7.9}$$

which is already non-trivial in $H^0(\gamma)$ and, actually, in $H^0(s)$

$$s a^{\mathrm{m-m}} = \gamma a_0^{\mathrm{m-m}}\left(\left[y^i\right]\right) = 0\,. \tag{7.10}$$

We will ask in addition that the derivative order of $a_0^{\mathrm{m-m}}$ does not exceed that of the original matter lagrangian $\mathcal{L}\left(\left[y^i\right]\right)$, which was assumed to be at most equal to two. Inserting (7.8) in (5.1) and using (7.10), we find that $a^{\mathrm{m-t}}$ itself must be a non-trivial element of $H^0(s|d)$

$$s a^{\mathrm{m-t}} = \partial_\mu m^\mu\,. \tag{7.11}$$

In order to solve the last equation, we proceed in the standard manner, namely, we develop $a^{\mathrm{m-t}}$ according to the antighost number up to a finite value

$$a^{\mathrm{m-t}} = \sum_{k=0}^{I} a_k^{\mathrm{m-t}}, \qquad \mathrm{agh}\left(a_k^{\mathrm{m-t}}\right) = k, \qquad \mathrm{gh}\left(a_k^{\mathrm{m-t}}\right) = 0, \qquad \varepsilon\left(a_k^{\mathrm{m-t}}\right) = 0, \tag{7.12}$$

such that (7.12) becomes equivalent to the tower of equations[10]

$$\gamma a_I^{\mathrm{m-t}} = 0, \tag{7.13}$$

$$\delta a_I^{\mathrm{m-t}} + \gamma a_{I-1}^{\mathrm{m-t}} = \partial_\mu \overset{(I-1)^\mu}{m}, \tag{7.14}$$

$$\delta a_k^{\mathrm{m-t}} + \gamma a_{k-1}^{\mathrm{m-t}} = \partial_\mu \overset{(k-1)^\mu}{m}, \qquad I-1 \geq k \geq 1, \tag{7.15}$$

where $\left(\overset{(k)^\mu}{m}\right)_{k=\overline{0,I}}$ are some local currents, with $\mathrm{agh}\left(\overset{(k)^\mu}{m}\right) = k$.

From (7.13) we read that the element of maximum antighost number, $a_I^{\mathrm{m-t}}$, belongs, as usually, to the non-trivial part of the cohomology $H(\gamma)$ of the overall "free" theory (7.1). The first two definitions in (7.5) show that the presence of the matter fields simply adds to the cohomology $H(\gamma)$ discussed in the subsection 5.1 the dependence on the fields y^i, on their antifields y_i^*, as well as on their space-time derivatives, such that the general solution to (7.13) is (up to irrelevant, γ-exact terms)

$$a_I^{\mathrm{m-t}} = \alpha_I^{\mathrm{m-t}}\left(\left[\Pi^{*\Delta}\right], \left[y_i^*\right], \left[F_{\lambda\mu\nu\xi|\alpha\beta}\right], \left[y^i\right]\right) \omega^I\left(C_\nu, \mathcal{F}_{\lambda\mu\nu\alpha}\right), \tag{7.16}$$

for all $I > 0$, where the notations $\Pi^{*\Delta}$ and ω^I have been introduced in the subsection 5.1 (see (5.10) and (5.26)). We observe that the matter sector contributes only to the invariant polynomials of the theory (7.1), while the elements ω^I are entirely due to the part corresponding to the tensor field $t_{\lambda\mu\nu|\alpha}$. Substituting (7.16) in (7.14), we conclude, like before, that a necessary condition for the existence of (non-trivial) solutions $a_{I-1}^{\mathrm{m-t}}$ is that $\alpha_I^{\mathrm{m-t}}$ are (non-trivial) objects from the local cohomology of the Koszul-Tate differential at antighost number $I > 0$ and zero pure ghost number. Meanwhile, $\alpha_I^{\mathrm{m-t}}$ must involve at least one matter generator in order to provide a corresponding lagrangian $a_0^{\mathrm{m-t}}$ that gives cross-couplings between y^i and $t_{\lambda\mu\nu|\alpha}$. Recalling once again the general theorems on the local cohomology of the Koszul-Tate differential together with the fact that the Cauchy order of the "free" theory (7.1) is equal to four, we obtain that (5.29) is valid also in this case. The result (5.30) is true and can be shown to hold like in the proof of theorem 3 from appendix B, and hence (5.31) is still checked. Moreover, we assumed that the Cauchy order of the matter theory alone is equal to one, such that the matter sector cannot be non-trivially involved in $H_I(\delta|d)$ and $H_I^{\mathrm{inv}}(\delta|d) = 0$ at $I > 1$. As a consequence, the non-trivial representatives of $\left(H_I(\delta|d)\right)_{I=2,3,4}$ and $\left(H_I^{\mathrm{inv}}(\delta|d)\right)_{I=2,3,4}$ are still covered by (5.32). Thus, we can always take $I \leq 4$ in (7.12) (see appendix C), but the cases $I = 4, 3, 2$ have already been studied in the subsections 5.2, 5.3 and respectively 5.4, where it was shown that there are no deformations. Anyway, even if we obtained non-trivial first-order deformations, they would describe only the self-interactions of the tensor field $t_{\lambda\mu\nu|\alpha}$.

[10] The fact that we can replace the equation $\gamma a_I^{\mathrm{m-t}} = \partial_\mu \overset{(I-1)^\mu}{m}$ with (7.13) is shown precisely like in the proof of corollary 1 from appendix A.

It is now clear that the cross-interactions between the tensor field $t_{\mu\nu|\alpha\beta}$ and the matter fields y^i at the first order in the coupling constant may be produced just by a first-order deformation of the master equation that stops at antighost number one, $I=1$.

7.3 The case $I = 1$

For $I=1$, the first-order deformation (7.12) decomposes into

$$a^{\mathrm{t-matt}} = a_1^{\mathrm{t-matt}} + a_0^{\mathrm{t-matt}}, \tag{7.17}$$

where $a_1^{\mathrm{t-matt}}$ ($\gamma a_1^{\mathrm{t-matt}} = 0$) is of the form (7.16) for $I=1$

$$a_1^{\mathrm{t-matt}} = \alpha_1^{\mathrm{t-matt}} \left(\left[t^{*\lambda\mu\nu|\alpha} \right], \left[y_i^* \right], \left[F_{\lambda\mu\nu\xi|\alpha\beta} \right], \left[y^i \right] \right) \omega^1 \left(\mathcal{F}_{\lambda\mu\nu\alpha} \right) + \gamma b_1^{\mathrm{t-matt}}, \tag{7.18}$$

and ω^1 are given in (5.66). Moreover, $a_1^{\mathrm{t-matt}}$ is subject to the equation

$$\delta a_1^{\mathrm{t-matt}} + \gamma a_0^{\mathrm{t-matt}} = \partial_\mu \overset{(0)}{m}{}^{\mu}, \tag{7.19}$$

so the invariant polynomial $\alpha_1^{\mathrm{t-matt}}$ is necessarily a non-trivial element from $H_1(\delta|d)$. Since $\alpha_1^{\mathrm{t-matt}}$ has the antighost number equal to one, it is linear in the antifields $t^{*\lambda\mu\nu|\alpha}$, y_i^*, as well as in their derivatives. We will initially discuss the restrictions on $\alpha_1^{\mathrm{t-matt}}$ imposed by the assumption on the maximum derivative order of $a_0^{\mathrm{t-matt}}$ being equal to two, and then analyze in detail the equation (7.19). First, $\alpha_1^{\mathrm{t-matt}}$ cannot involve either the curvature $F_{\lambda\mu\nu\xi|\alpha\beta}$ or its derivatives and, along the same line, the second- or higher-order derivatives of the matter fields. Second, the eligible terms containing the objects $\left[t^{*\lambda\mu\nu|\alpha} \right]$ cannot involve the derivatives of the antifields $t^{*\lambda\mu\nu|\alpha}$ and may depend at most on the undifferentiated matter fields y^i. Third, the admitted pieces involving the quantities $[y_i^*]$ may be of two types: ones are proportional with y_i^* and the others with their first-order derivatives.[11] We can always move the derivatives from y_i^* in the latter type of components by performing an integration by parts in the corresponding functionals. Ignoring the divergences, taking into account the fact that the first-order derivatives of ω^1 are γ-exact and that any polynomial in $\left[y^i \right]$ is γ-closed, we reach precisely the former type of terms, linear in the undifferentiated antifields y_i^*, so the pieces containing the first-order derivatives of the antifields y_i^* can be discarded from $\alpha_1^{\mathrm{t-matt}}$. In consequence, the eligible objects from $\alpha_1^{\mathrm{t-matt}}$ implying $[y_i^*]$ contain the undifferentiated antifields y_i^* and may depend on y^i and ∂y^i (with the precaution from the footnote 11). Under these circumstances, we have that

$$\begin{aligned} a_1^{\mathrm{t-matt}} &= f^{\lambda\mu\nu\alpha\lambda'\mu'\nu'\alpha'} \left(y^i \right) t^*_{\lambda\mu\nu|\alpha} \mathcal{F}_{\lambda'\mu'\nu'\alpha'} + y_i^* g^i_{\lambda\mu\nu\alpha} \left(y^j, \partial y^j \right) \mathcal{F}^{\lambda\mu\nu\alpha} + \\ &\quad + \gamma b_1^{\mathrm{t-matt}} \\ &\equiv a_1'^{\mathrm{t-matt}} + a_1''^{\mathrm{t-matt}}, \end{aligned} \tag{7.20}$$

where we used the notations

$$a_1'^{\mathrm{t-matt}} = f^{\lambda\mu\nu\alpha\lambda'\mu'\nu'\alpha'} \left(y^i \right) t^*_{\lambda\mu\nu|\alpha} \mathcal{F}_{\lambda'\mu'\nu'\alpha'} + \gamma b_1'^{\mathrm{t-matt}}, \tag{7.21}$$

$$a_1''^{\mathrm{t-matt}} = y_i^* g^i_{\lambda\mu\nu\alpha} \left(y^j, \partial y^j \right) \mathcal{F}^{\lambda\mu\nu\alpha} + \gamma b_1''^{\mathrm{t-matt}}, \tag{7.22}$$

$$b_1^{\mathrm{t-matt}} = b_1'^{\mathrm{t-matt}} + b_1''^{\mathrm{t-matt}}. \tag{7.23}$$

[11] The first-order derivatives of the matter fields are allowed to enter $a_1^{\mathrm{t-matt}}$ only if the matter theory is first-order in the derivatives.

The function $f^{\lambda\mu\nu\alpha\lambda'\mu'\nu'\alpha'}$ is bosonic, while $g^i_{\lambda\mu\nu\alpha}$ possesses the Grassmann parity ε_i and, moreover, $f^{\lambda\mu\nu\alpha\lambda'\mu'\nu'\alpha'}$ must effectively involve the matter fields y^i, because otherwise the corresponding $a_1'^{\text{t-matt}}$ cannot lead to cross-couplings.

The term $a_1'^{\text{t-matt}}$, if consistently lifted to antighost number zero, would produce a lagrangian $a_0'^{\text{t-matt}}$ that is quadratic in the tensor field $t_{\lambda\mu\nu|\alpha}$, while $a_1''^{\text{t-matt}}$ would give an $a_0''^{\text{t-matt}}$ that is linear in the same field, and hence $a_1'^{\text{t-matt}}$ and $a_1''^{\text{t-matt}}$ contribute to different types of vertices. Therefore, the equation (7.19) is equivalent to two equations, that must hold independently

$$\delta a_1'^{\text{t-matt}} + \gamma a_0'^{\text{t-matt}} = \partial_\mu \overset{(0)}{m}{}'^\mu , \tag{7.24}$$

$$\delta a_1''^{\text{t-matt}} + \gamma a_0''^{\text{t-matt}} = \partial_\mu \overset{(0)}{m}{}''^\mu . \tag{7.25}$$

Related to (7.21), we note that the identity (3.3) prevents the tensors f_1 and f_2 to contain Levi-Civita symbols contracted on all the indices of $t^*_{\lambda\mu\nu|\alpha}$, such that we remain with the following independent possibility in $D = 6$ dimensions

$$a_1'^{(D=6)} = \sigma^{\alpha\beta} \varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'} f\left(y^i\right) t^*_{\lambda\mu\nu|\alpha} \mathcal{F}_{\lambda'\mu'\nu'\beta} + \gamma b_1'^{(D=6)} . \tag{7.26}$$

In the above f is a bosonic function of the undifferentiated matter fields. Proceeding in a manner similar to that from the subsection 5.5 we find that (7.26) leads to a consistent $a_0'^{(D=6)}$ if and only if it is γ-trivial, i.e., $f\left(y^i\right) = 0$.

As a consequence, we can always set

$$a_1'^{\text{t-matt}} = 0 \tag{7.27}$$

in (7.20), such that the non-trivial cross-couplings between the tensor field $t_{\lambda\mu\nu|\alpha}$ and the matter fields y^i may be obtained starting from an antighost number one component of the first-order deformation which can only be linear in the matter antifields

$$a_1^{\text{t-matt}} = a_1''^{\text{t-matt}} = y_i^* g^i_{\lambda\mu\nu\alpha}\left(y^j, \partial y^j\right) \mathcal{F}^{\lambda\mu\nu\alpha} , \tag{7.28}$$

where $a_1''^{\text{t-matt}}$ must obey the equation (7.25) (we chose $b_1''^{\text{t-matt}} = 0$ in (7.22)). By direct computation, from (7.22) and (7.5) we get that

$$\delta a_1''^{\text{t-matt}} = \frac{\delta^R \mathcal{L}}{\delta y^i} g^i_{\lambda\mu\nu\alpha}\left(y^j, \partial y^j\right) \mathcal{F}^{\lambda\mu\nu\alpha} , \tag{7.29}$$

where the notation $\delta^R \mathcal{L}/\delta y^i$ signifies the right Euler-Lagrange derivatives with respect to y^i. Comparing (7.29) with (7.25) and recalling the formula (5.18), we can state that (7.28) is consistent at antighost number zero if and only if there exists a bosonic current $j^\rho_{\lambda\mu\nu\alpha}$ such that

$$\frac{\delta^R \mathcal{L}}{\delta y^i} g^i_{\lambda\mu\nu\alpha}\left(y^j, \partial y^j\right) = \partial_\rho j^\rho_{\lambda\mu\nu\alpha}\left(y^j, \partial y^j\right) . \tag{7.30}$$

The above relation is nothing but Noether's theorem for the matter theory (7.2) corresponding to a rigid symmetry whose parameter is a bosonic, completely antisymmetric and constant tensor $\xi^{\lambda\mu\nu\alpha}$

$$\Delta y^i = g^i_{\lambda\mu\nu\alpha}\left(y^j, \partial y^j\right) \xi^{\lambda\mu\nu\alpha} , \qquad \varepsilon\left(\xi^{\lambda\mu\nu\alpha}\right) = 0 . \tag{7.31}$$

The equation (7.30) implements the necessary condition that the invariant polynomial from (7.28), namely,

$$\alpha^{i}_{1\lambda\mu\nu\alpha} = y^{*}_{i} g^{i}_{\lambda\mu\nu\alpha}\left(y^{j}, \partial y^{j}\right), \tag{7.32}$$

belongs to $H_1(\delta|d)$ and exhibits the natural relationship between the elements of $H_1(\delta|d)$ and the global symmetries of the theory (7.1). However, in agreement with the general line developed in sections 5 and 6, $\alpha^{i}_{1\lambda\mu\nu\alpha}$ must be a non-trivial object from $H_1(\delta|d)$, which is equivalent to the fact that there exist non-trivial global symmetries (7.31), and therefore non-trivial conserved currents (7.30). The form of a trivial global symmetry of the matter theory reads

$$\Delta y^{i} = \frac{\delta^{R} L}{\delta y^{j}} g^{ji}_{A} \xi^{A}, \qquad g^{ji}_{A} = -\left(-\right)^{\varepsilon_i \varepsilon_j} g^{ij}_{A}, \tag{7.33}$$

where ξ^{A} are the associated rigid parameters, such that Noether's theorem becomes

$$\frac{\delta^{R} L}{\delta y^{i}} \frac{\delta^{R} L}{\delta y^{j}} g^{ji}_{A} = \partial_{\mu} j^{\mu}_{A} \equiv 0, \tag{7.34}$$

and so the identically conserved currents j^{μ}_{A} are also trivial

$$j^{\mu}_{A} = \partial_{\nu} k^{\nu\mu}_{A}, \qquad k^{\nu\mu}_{A} = -k^{\mu\nu}_{A}. \tag{7.35}$$

Assuming that the matter theory indeed possesses a non-trivial rigid symmetry of the form (7.31), with the help of the relations (7.29)–(7.30) and (5.18) we find the solution to the equation (7.25) like

$$a^{\mathrm{t-matt}}_{0} = a''^{\mathrm{t-matt}}_{0} = \frac{1}{3} j^{\rho}_{\lambda\mu\nu\alpha}\left(y^{j}, \partial y^{j}\right) \partial^{[\lambda} t^{\mu\nu\alpha]|}{}_{\rho}. \tag{7.36}$$

Putting together the formulas (7.28) and (7.36), we can write that the general expression of the first-order deformation (7.17) reads as

$$a^{\mathrm{t-matt}} = y^{*}_{i} g^{i}_{\lambda\mu\nu\alpha}\left(y^{j}, \partial y^{j}\right) \mathcal{F}^{\lambda\mu\nu\alpha} + \frac{1}{3} j^{\rho}_{\lambda\mu\nu\alpha}\left(y^{j}, \partial y^{j}\right) \partial^{[\lambda} t^{\mu\nu\alpha]|}{}_{\rho}. \tag{7.37}$$

(In principle, one can always add to (7.37) quantities $\bar{a}^{\mathrm{t-matt}}_{0}\left(\left[t_{\lambda\mu\nu|\alpha}\right], \left[y^{i}\right]\right)$ that are γ-closed modulo d, $\gamma\bar{a}^{\mathrm{t-matt}}_{0}\left(\left[t_{\lambda\mu\nu|\alpha}\right], \left[y^{i}\right]\right) = \partial_{\mu} p^{\mu}$. They are irrelevant anyway since they do not deform the original gauge transformations, but simply add gauge-invariant terms to the deformed lagrangian action. However, taking into account the supplementary conditions that must be satisfied by $\bar{a}^{\mathrm{t-matt}}_{0}$, like those on the maximum derivative order, on Poincaré invariance and on yielding effective cross-couplings, we find no suitable solutions $\bar{a}^{\mathrm{t-matt}}_{0}$ in $D \geq 5$ space-time dimensions, so (7.37) indeed represents the most general expression of the first-order deformation related to the cross-couplings between the tensor field $t_{\lambda\mu\nu|\alpha}$ and the matter fields y^{i}.)

So far we have shown that there exist non-trivial deformations at the first order in the coupling constant describing cross-interactions between the tensor field $t_{\lambda\mu\nu|\alpha}$ and the matter fields y^{i} if and only if the matter theory displays a non-trivial global symmetry of the form (7.31).

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7.4 Higher-order deformations

Combining (7.9) with (7.37), we obtained that the first-order deformation of the solution to the master equation for the theory (7.1) is

$$S_1 = \int d^D x \Big(a_0^{\mathrm{m-m}} \left([y^i] \right) + y_i^* g^i_{\lambda\mu\nu\alpha} \left(y^j, \partial y^j \right) \mathcal{F}^{\lambda\mu\nu\alpha} + \\ + \frac{1}{3} j^\rho_{\lambda\mu\nu\alpha} \left([y^i] \right) \partial^{[\lambda} t^{\mu\nu\alpha]|}{}_{\rho} \Big). \tag{7.38}$$

Next, we investigate the equation (4.5), which governs the second-order deformation. If we make the notations $S_2 = \int d^D x \, b$ and $\frac{1}{2}(S_1, S_1) = \int d^D x \, \theta$, then the equation (4.5) takes the local form

$$\theta = -sb + \partial_\mu p^\mu \, . \tag{7.39}$$

By direct computation we find that

$$\theta = \left(\frac{1}{3} \partial^{[\lambda'} t^{\mu'\nu'\alpha']|}{}_{\rho} \frac{\delta^R j^\rho_{\lambda'\mu'\nu'\alpha'}}{\delta y^i} + \frac{\delta^R a_0^{\mathrm{m-m}}}{\delta y^i} \right) g^i_{\lambda\mu\nu\alpha} \mathcal{F}^{\lambda\mu\nu\alpha} + \\ + \frac{1}{2} y_i^* \left(\frac{\delta^R g^i_{\lambda\mu\nu\alpha}}{\delta y^j} g^j_{\lambda'\mu'\nu'\alpha'} - \frac{\delta^R g^i_{\lambda'\mu'\nu'\alpha'}}{\delta y^j} g^j_{\lambda\mu\nu\alpha} \right) \mathcal{F}^{\lambda\mu\nu\alpha} \mathcal{F}^{\lambda'\mu'\nu'\alpha'} \, . \tag{7.40}$$

We observe that second term from the right-hand side of (7.40) cannot be written under a s-exact modulo d form,[12] as required by (7.39), and therefore it must vanish

$$\frac{\delta^R g^i_{\lambda\mu\nu\alpha}}{\delta y^j} g^j_{\lambda'\mu'\nu'\alpha'} - \frac{\delta^R g^i_{\lambda'\mu'\nu'\alpha'}}{\delta y^j} g^j_{\lambda\mu\nu\alpha} = 0 \, . \tag{7.41}$$

The above condition (7.41) can be satisfied only in $D = 6$ space-time dimensions, where its solution depends on the space-time co-ordinates

$$g^i_{\lambda\mu\nu\alpha} = c \varepsilon_{\lambda\mu\nu\alpha\rho\sigma} x^{[\rho} \partial^{\sigma]} y^i \, , \tag{7.42}$$

with c an arbitrary real constant. Replacing (7.42) in (7.31) and making the notation

$$\varepsilon_{\lambda\mu\nu\alpha\rho\sigma} \xi^{\lambda\mu\nu\alpha} \equiv \xi_{\rho\sigma} \, , \qquad \xi_{\rho\sigma} = -\xi_{\rho\sigma} \, , \tag{7.43}$$

we get that (7.42) corresponds to the rigid symmetry

$$\Delta y^i = c x^{[\rho} \partial^{\sigma]} y^i \xi_{\rho\sigma} \, , \tag{7.44}$$

of the matter action. The associated conserved current will also involve the space-time co-ordinates, and hence the deformed lagrangian (7.36) would break the required Poincaré invariance of the cross-interactions, so we must remove the piece (7.37) from (7.38) by setting $c = 0$. In this situation the first-order deformation (7.38) reduces to

$$S_1 = \int d^D x \, a_0^{\mathrm{m-m}} \left([y^i] \right) , \tag{7.45}$$

[12]This is because: (1) it cannot reduce to a divergence; (2) it is a non-trivial element from $H^2(\gamma)$ of antighost number one; (3) it cannot be δ-exact unless the global symmetry (7.31) is trivial (see (7.33)).

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and it is already consistent to all higher-orders in the coupling constant, in the sense that the equations (4.5), etc., are fulfilled with the choice

$$S_k = 0\,, \qquad k > 1\,. \tag{7.46}$$

Thus, the deformation procedure does not modify the gauge transformations (7.3) of the action (7.1), but simply adds arbitrary terms to the matter lagrangian (type (i) interactions).

The conclusion of this section is that, under the simultaneous requirements of Lorentz covariance, Poincaré invariance and space-time locality of the deformations, combined with the requirement on the maximum derivative order of the interacting lagrangian being equal to two, there are no consistent and non-trivial cross-interactions between the mixed symmetry-type tensor field $t_{\lambda\mu\nu|\alpha}$ and matter fields in $D \geq 5$ dimensions.

8. Conclusion

To conclude with, in this paper we have used the powerful reformulation of interactions in gauge theories by means of local BRST cohomology and proved that the free massless tensor field $t_{\lambda\mu\nu|\alpha}$, which is known to be dual to the Pauli-Fierz theory in $D = 6$ dimensions, admits no smooth, local, consistent self-interactions and, in the meantime, cannot be coupled either to the massless spin-two field or to matter fields. Our analysis was constantly based on the assumptions that the maximum derivative order of the interacting lagrangian is equal to two and that the deformations are Lorentz-covariant and Poincaré-invariant. It is possible that the relaxation of the condition on the derivative order allows non-trivial consistent interactions, at least with the massless spin-two field. We believe that the approach to this matter in the case where the derivative assumption is dropped out would simplify if one worked in the first-order formulation [13, 14] of such an "exotic" tensor field, instead of the second-order one used in the present paper, and also employed a tetrad-like description for the free limit of linearized General Relativity.

A. Cohomology of γ in the space of invariant polynomials

Theorem 1. *The cohomology of d in form degree strictly less than D is trivial in the space of invariant polynomials with strictly positive antighost number. This means that the conditions*

$$\gamma\alpha = 0\,, \qquad d\alpha = 0\,, \qquad \mathrm{agh}\,(\alpha) > 0\,, \qquad \deg\alpha < D\,, \qquad \alpha = \alpha\left(\left[\Pi^{*\Delta}\right], [F]\right), \tag{A.1}$$

imply

$$\alpha = d\beta, \tag{A.2}$$

for some invariant polynomial $\beta\left(\left[\Pi^{*\Delta}\right], [F]\right)$.

Proof. The proof is standard and can be done like in [30]. □

In form degree D the theorem 1 is replaced with: let $\alpha = \rho dx^0 \wedge \cdots \wedge dx^{D-1}$ be a d-exact invariant polynomial of form degree D and of strictly positive antighost number,

$\mathrm{agh}\,(\alpha) = m > 0$, $\deg\,(\alpha) = D$, $\alpha = d\beta$. Then, one can take the $(D-1)$-form β to be an invariant polynomial (of antighost number m). In dual notations, this means that if ρ with $\mathrm{agh}\,(\rho) = m > 0$ is an invariant polynomial whose Euler-Lagrange derivatives are all vanishing, $\rho = \partial_\mu j^\mu$, then j^μ can be taken to be also invariant. Theorem 1 can be generalized as follows.

Theorem 2. *The cohomology of d computed in $H\,(\gamma)$ is trivial in form degree strictly less than D and in strictly positive antighost number*

$$H^{g,m}_k\,(d, H\,(\gamma)) = 0\,, \qquad m > 0\,, \qquad k < D\,, \tag{A.3}$$

where k is the form degree, m is the antighost number and g is the ghost number.

Proof. The proof is standard and can be done like in [26], theorem 5.2. □

Theorem 2 is one of the main tools needed for the computation of $H\,(s|d)$. In particular, it implies that there is no non-trivial descent for $H\,(\gamma|d)$ in strictly positive antighost number.

Corollary 1. *If a with*

$$\mathrm{agh}\,(a) = m > 0\,, \qquad \mathrm{gh}\,(a) = g \geq -m\,, \qquad \deg\,(a) = k \leq D, \tag{A.4}$$

satisfies the equation

$$\gamma a + db = 0, \tag{A.5}$$

where

$$\mathrm{agh}\,(b) = m > 0\,, \qquad \mathrm{gh}\,(b) = g+1\,, \qquad \deg\,(b) = k-1 < D\,, \tag{A.6}$$

then one can always redefine a

$$a \to a' = a + d\nu\,, \tag{A.7}$$

so that

$$\gamma a' = 0\,. \tag{A.8}$$

Proof. We construct the descent associated with the equation (A.5). Acting with γ on (A.5), we find that

$$d\,(-\gamma b) = 0\,, \tag{A.9}$$

such that the triviality of the cohomology of d implies that

$$\gamma b + dc = 0\,, \tag{A.10}$$

where

$$\mathrm{agh}\,(c) = m > 0\,, \qquad \mathrm{gh}\,(c) = g+2\,, \qquad \deg\,(c) = k-2\,. \tag{A.11}$$

Going on in the same way, we get the next equation from the descent

$$\gamma c + de = 0\,, \tag{A.12}$$

with

$$\mathrm{agh}\,(e) = m > 0\,, \qquad \mathrm{gh}\,(e) = g + 3\,, \qquad \deg\,(e) = k - 3\,, \tag{A.13}$$

and so on. The descent stops after a finite number of steps with the last equations

$$\gamma t + du = 0\,, \tag{A.14}$$

$$\gamma u + dv = 0\,, \tag{A.15}$$

$$\gamma v = 0\,, \tag{A.16}$$

either because v is a zero-form or because we stopped at a higher form-degree with a γ-closed term. It is essential to remark that irrespective of the step at which the descent is cut, we have that

$$\mathrm{agh}\,(v) = m > 0\,, \qquad \mathrm{gh}\,(v) = g' > m\,, \qquad \deg\,(v) = k' < D\,. \tag{A.17}$$

(The earliest step where the descent may terminate is $v = b$ and, according to (A.6), we have that $\deg\,(b) = k - 1 < D$ and $\mathrm{gh}\,(b) = g + 1 > m$.)

The equations (A.15)–(A.16) together with the conditions (A.17) tell us that v belongs to $H_{k'}^{g',m}\,(d, H\,(\gamma))$ for $m > 0$, $k' < D$ and $g' > m$, so theorem 2 guarantees that v is trivial in $H_{k'}^{g',m}\,(d, H\,(\gamma))$

$$v = d\nu' + \gamma\rho'\,, \qquad \gamma\nu' = 0\,, \tag{A.18}$$

which[13] substituted in (A.15) allows us, due to the anticommutation between d and γ, to replace it with the equivalent equation

$$\gamma u' = 0\,, \tag{A.19}$$

where

$$u' = u - d\rho'\,. \tag{A.20}$$

In the meantime, (A.20) and the nilpotency of d induces that $du' = du$, such that the equation (A.14) becomes $\gamma t + du' = 0$. Reprising the same argument in relation with (A.19) and the last equation, we find that it can be replaced with $\gamma t' = 0$, where $t' = t - d\rho''$ and ρ'' comes from $u' = d\nu'' + \gamma\rho''$, with $\gamma\nu'' = 0$. Performing exactly the same operations for the remaining equations from the descent, we finally infer that (A.5) is equivalent with $\gamma a' = 0$, where $a' = a - d\rho'''$ and ρ''' appears in $b' = d\nu''' + \gamma\rho'''$, with $\gamma\nu''' = 0$. The corollary is now demonstrated once we perform the obvious identification $\nu = -\rho'''$. $\square$

B. Some results on the invariant cohomology $H^{\mathrm{inv}}\,(\delta|d)$

Here, we prove that if an invariant polynomial α_I is trivial in $H_I\,(\delta|d)$, then it can be taken to be trivial also in $H_I^{\mathrm{inv}}\,(\delta|d)$. We consider only the case $I \geq 4$ since our main scope is to argue the triviality of $H^{\mathrm{inv}}\,(\delta|d)$ in antighost number strictly greater than four. First, we prove the following lemma.

[13]Note that if the descent stops in form degree zero, $\deg\,(v) = 0$, then the proof remains valid with the sole modification $\nu' = 0$ in (A.18).

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Lemma 1. *Let α be a δ-exact invariant polynomial*

$$\alpha = \delta\beta\,. \tag{B.1}$$

Then, β can also be taken to be an invariant polynomial.

Proof. Let v be a function of $\left[\Pi^{*\Delta}\right]$ and $\left[t_{\lambda\mu\nu|\alpha}\right]$. The dependence of v on $\left[t_{\lambda\mu\nu|\alpha}\right]$ can be reorganized as a dependence on the curvature and its derivatives, $[F]$, and on $\left\{\tilde{t}_{\lambda\mu\nu|\alpha}\right\} = \left\{t_{\lambda\mu\nu|\alpha}, \partial t_{\lambda\mu\nu|\alpha}, \cdots\right\}$, where $\tilde{t}_{\lambda\mu\nu|\alpha}$ are not γ-invariant. If v is γ-invariant, then it does not involve $\left\{\tilde{t}_{\lambda\mu\nu|\alpha}\right\}$, i.e., $v = v|_{\tilde{t}_{\lambda\mu\nu|\alpha}=0}$, so we have by hypothesis that

$$\alpha = \alpha|_{\tilde{t}_{\lambda\mu\nu|\alpha}=0}\,. \tag{B.2}$$

On the other hand, β depends in general on $\left[\Pi^{*\Delta}\right]$, $[F]$ and $\left\{\tilde{t}_{\lambda\mu\nu|\alpha}\right\}$. Making $\tilde{t}_{\lambda\mu\nu|\alpha} = 0$ in (B.1), using (B.2) and taking into account the fact that δ commutes with the operation of setting $\tilde{t}_{\lambda\mu\nu|\alpha}$ equal to zero, we find that

$$\alpha = \delta\left(\beta|_{\tilde{t}_{\lambda\mu\nu|\alpha}=0}\right), \tag{B.3}$$

with $\beta|_{\tilde{t}_{\lambda\mu\nu|\alpha}=0}$ invariant. This proves the lemma. □

Now, we have the necessary tools for proving the next theorem.

Theorem 3. *Let α_k^p be an invariant polynomial with $\deg\left(\alpha_k^p\right) = p$ and $\mathrm{agh}\left(\alpha_k^p\right) = k$, which is δ-exact modulo d*

$$\alpha_k^p = \delta\lambda_{k+1}^p + d\lambda_k^{p-1}\,, \qquad k \geq 4\,. \tag{B.4}$$

Then, we can choose λ_{k+1}^p and λ_k^{p-1} to be invariant polynomials.

Proof. Initially, by successively acting with d and δ on (B.4) we obtain a tower of equations of the same type. Indeed, acting with d on (B.4) we find that $d\alpha_k^p = -\delta\left(d\lambda_{k+1}^p\right)$. On the other hand, as $d\alpha_k^p$ is invariant, by means of lemma 1 we obtain that $d\alpha_k^p = -\delta\alpha_{k+1}^{p+1}$, with α_{k+1}^{p+1} invariant. From the last two relations we deduce that $\delta(\alpha_{k+1}^{p+1} - d\lambda_{k+1}^p) = 0$. As δ is acyclic at strictly positive antighost numbers, the last relation implies that

$$\alpha_{k+1}^{p+1} = \delta\lambda_{k+2}^{p+1} + d\lambda_{k+1}^p\,. \tag{B.5}$$

Starting now with (B.5) and reprising the same operations like those performed between the formulas (B.4) and (B.5), we obtain a descent that stops in form degree D with the equation $\alpha_{k+D-p}^D = \delta\lambda_{k+D-p+1}^D + d\lambda_{k+D-p}^{D-1}$. Now, we act with δ on (B.4) and deduce that $\delta\alpha_k^p = -d\delta\lambda_k^{p-1}$. As $\delta\alpha_k^p$ is invariant and of strictly positive antighost number, due to the theorem 1 we obtain that $\delta\alpha_k^p = -d\alpha_{k-1}^{p-1}$, where α_{k-1}^{p-1} is invariant. Using the last two relations we get that $d(\alpha_{k-1}^{p-1} - \delta\lambda_k^{p-1}) = 0$, such that it follows

$$\alpha_{k-1}^{p-1} = \delta\lambda_k^{p-1} + d\lambda_{k-1}^{p-2}\,. \tag{B.6}$$

If $k = 4$, we cannot go down since by assumption $k \geq 4$, and so the bottom of the tower is (B.4) for $k = 4$. Starting from (B.6) and reprising the same procedure we reach a descent that ends at either form degree zero or antighost number four, hence the last equation respectively takes the form

$$\alpha^0_{k-p} = \delta\lambda^0_{k-p+1}\,, \tag{B.7}$$

or

$$\alpha^{p-k+4}_4 = \delta\lambda^{p-k+4}_5 + d\lambda^{p-k+3}_4\,. \tag{B.8}$$

In consequence, the procedure described in the above leads to the chain

$$\begin{aligned}
\alpha^D_{k+D-p} &= \delta\lambda^D_{k+D-p+1} + d\lambda^{D-1}_{k+D-p}\,,\\
&\;\;\vdots\\
\alpha^{p+1}_{k+1} &= \delta\lambda^{p+1}_{k+2} + d\lambda^{p}_{k+1}\,,\\
\alpha^{p}_{k} &= \delta\lambda^{p}_{k+1} + d\lambda^{p-1}_{k}\,,\\
\alpha^{p-1}_{k-1} &= \delta\lambda^{p-1}_{k} + d\lambda^{p-2}_{k-1}\,,\\
&\;\;\vdots\\
\alpha^0_{k-p} &= \delta\lambda^0_{k-p+1} \qquad \text{or} \qquad \alpha^{p-k+4}_4 = \delta\lambda^{p-k+4}_5 + d\lambda^{p-k+3}_4\,.
\end{aligned} \tag{B.9}$$

All the α's in the descent (B.9) are invariant.

Now, we show that if one of the λ's in (B.9) is invariant, then all the other λ's can be taken to be also invariant. Indeed, let λ^{A-1}_B be invariant. It is involved in two of the equations from (B.9), namely

$$\alpha^A_B = \delta\lambda^A_{B+1} + d\lambda^{A-1}_B\,, \tag{B.10}$$

$$\alpha^{A-1}_{B-1} = \delta\lambda^{A-1}_B + d\lambda^{A-2}_{B-1}\,. \tag{B.11}$$

From (B.10) it results that $\alpha^A_B - d\lambda^{A-1}_B$ is invariant. Then, in agreement with lemma 1 the object λ^A_{B+1} can be chosen to be invariant. Using (B.11), we have that $\alpha^{A-1}_{B-1} - \delta\lambda^{A-1}_B$ is invariant, such that theorem 1 ensures that λ^{A-2}_{B-1} is also invariant. On the other hand, λ^A_{B+1} and λ^{A-2}_{B-1} are involved in other two sets of equations from the descent. (For instance, the former element appears in the equations $\alpha^{A+1}_{B+1} = \delta\lambda^{A+1}_{B+2} + d\lambda^A_{B+1}$ and $\alpha^{A+2}_{B+2} = \delta\lambda^{A+2}_{B+3} + d\lambda^{A+1}_{B+2}$.) Going on in the same fashion, we find that all the λ's are invariant. In the case where λ^{A-1}_B appears at the top or at the bottom of the descent, we act in a similar way, but only with respect to a single equation. The above considerations emphasize that it is enough to verify the theorem in form degree D.

If $k > D$, the last equation from the descent (B.9) for $p = D$ reads as

$$\alpha^0_{k-D} = \delta\lambda^0_{k-D+1}\,. \tag{B.12}$$

Using lemma 1, it results that λ^0_{k-D+1} can be taken to be invariant, such that all the λ's from the descent can be chosen invariant.

In the sequel we consider the case $4 \leq k \leq D$. Here we will use an inductive procedure to be made clear below. The top equation from (B.9) for $p = D$, written in dual notations, takes the form

$$\alpha_k = \delta\lambda_{k+1} + \partial_\mu\lambda^\mu_k\,, \qquad k \geq 4\,. \tag{B.13}$$

On the other hand, we can express α_k in terms of its Euler-Lagrange (E.L.) derivatives by means of the homotopy formula

$$\alpha_k = \int_0^1 d\tau \left(\frac{\delta^R \alpha_k}{\delta C^*_\alpha}(\tau)\, C^*_\alpha + \frac{\delta^R \alpha_k}{\delta {\mathcal{C}'}^*_{\alpha\beta}}(\tau)\, {\mathcal{C}'}^*_{\alpha\beta} + \frac{\delta^R \alpha_k}{\delta {\mathcal{G}'}^*_{\mu\nu|\alpha}}(\tau)\, {\mathcal{G}'}^*_{\mu\nu|\alpha} + \right.$$
$$\left. + \frac{\delta^R \alpha_k}{\delta t^*_{\mu\nu\lambda|\alpha}}(\tau)\, t^*_{\mu\nu\lambda|\alpha} + \frac{\delta^R \alpha_k}{\delta t_{\mu\nu\lambda|\alpha}}(\tau)\, t_{\mu\nu\lambda|\alpha} \right) + \partial_\mu j^\mu_k, \tag{B.14}$$

where $\frac{\delta^R \alpha_k}{\delta C^*_\alpha}(\tau) = \frac{\delta^R \alpha_k}{\delta C^*_\alpha}\left(\tau\left[t_{\mu\nu\lambda|\alpha}\right], \tau\left[\Pi^{*\Delta}\right]\right)$ and similarly for the other terms. Denoting the E.L. derivatives of λ_{k+1} by

$$\frac{\delta^R \lambda_{k+1}}{\delta C^*_\alpha} = G^\alpha_{k-3}\,, \qquad \frac{\delta^R \lambda_{k+1}}{\delta {\mathcal{C}'}^*_{\alpha\beta}} = K^{\alpha\beta}_{k-2}\,, \qquad \frac{\delta^R \lambda_{k+1}}{\delta {\mathcal{G}'}^*_{\mu\nu|\alpha}} = M^{\mu\nu|\alpha}_{k-1}\,, \tag{B.15}$$

$$\frac{\delta^R \lambda_{k+1}}{\delta t^*_{\mu\nu\lambda|\alpha}} = N^{\mu\nu\lambda|\alpha}_k, \qquad \frac{\delta^R \lambda_{k+1}}{\delta t_{\mu\nu\lambda|\alpha}} = L^{\mu\nu\lambda|\alpha}_{k+1}, \tag{B.16}$$

and using (B.13) we find after some computation that the E.L. derivatives of α_k are given by

$$\frac{\delta^R \alpha_k}{\delta C^*_\alpha} = \delta G^\alpha_{k-3}\,, \qquad \frac{\delta^R \alpha_k}{\delta {\mathcal{C}'}^*_{\alpha\beta}} = -\delta K^{\alpha\beta}_{k-2} - 6\partial^\alpha G^\beta_{k-3}\,, \tag{B.17}$$

$$\frac{\delta^R \alpha_k}{\delta {\mathcal{G}'}^*_{\mu\nu|\alpha}} = \delta M^{\mu\nu|\alpha}_{k-1} - \partial^{[\mu} K^{\nu]\alpha}_{k-2}\,, \qquad \frac{\delta^R \alpha_k}{\delta t^*_{\mu\nu\lambda|\alpha}} = -\delta N^{\mu\nu\lambda|\alpha}_k + \partial^{[\mu} M^{\nu\lambda]\alpha}_{k-1}\,, \tag{B.18}$$

$$\begin{aligned}\frac{\delta^R \alpha_k}{\delta t_{\mu\nu\lambda|\alpha}} &= \delta L^{\mu\nu\lambda|\alpha}_{k+1} + \partial^\rho \partial_\rho N^{\lambda\mu\nu|\alpha}_k - \partial_\rho \left(\partial^{[\lambda} N^{\mu\nu]\rho|\alpha}_k + \partial^\alpha N^{\lambda\mu\nu|\rho}_k\right) + \\ &\quad + \partial^\alpha \partial^{[\lambda} N^{\mu\nu]\rho|\tau}_k \sigma_{\rho\tau} + \sigma^{\alpha[\lambda} \partial_\rho \left(\partial_\beta N^{\mu\nu]\rho|\beta}_k - \partial^\mu N^{\nu]\rho\sigma|\tau}_k \sigma_{\sigma\tau}\right) - \\ &\quad - \sigma^{\alpha[\lambda} \partial^\rho \partial_\rho N^{\mu\nu]\sigma|\tau}_k \sigma_{\sigma\tau}.\end{aligned} \tag{B.19}$$

As the E.L. derivatives of an invariant quantity are also invariant, the former equation in (B.17) together with lemma 1 lead to

$$\frac{\delta^R \alpha_k}{\delta C^*_\alpha} = \delta \bar{G}^\alpha_{k-3}\,, \tag{B.20}$$

with $\bar{G}^\alpha_{k-3}$ invariant. Following a similar reasoning, we find that

$$\frac{\delta^R \alpha_k}{\delta {\mathcal{C}'}^*_{\alpha\beta}} = -\delta \bar{K}^{\alpha\beta}_{k-2} - 6\partial^\alpha \bar{G}^\beta_{k-3}\,, \tag{B.21}$$

$$\frac{\delta^R \alpha_k}{\delta {\mathcal{G}'}^*_{\mu\nu|\alpha}} = \delta \bar{M}^{\mu\nu|\alpha}_{k-1} - \partial^{[\mu} \bar{K}^{\nu]\alpha}_{k-2}\,, \qquad \frac{\delta^R \alpha_k}{\delta t^*_{\mu\nu\lambda|\alpha}} = -\delta \bar{N}^{\mu\nu\lambda|\alpha}_k + \partial^{[\mu} \bar{M}^{\nu\lambda]\alpha}_{k-1}\,, \tag{B.22}$$

$$\begin{aligned}\frac{\delta^R \alpha_k}{\delta t_{\mu\nu\lambda|\alpha}} &= \delta \bar{L}^{\mu\nu\lambda|\alpha}_{k+1} + \partial^\rho \partial_\rho \bar{N}^{\lambda\mu\nu|\alpha}_k - \partial_\rho \left(\partial^{[\lambda} \bar{N}^{\mu\nu]\rho|\alpha}_k + \partial^\alpha \bar{N}^{\lambda\mu\nu|\rho}_k\right) + \\ &\quad + \partial^\alpha \partial^{[\lambda} \bar{N}^{\mu\nu]\rho|\tau}_k \sigma_{\rho\tau} + \sigma^{\alpha[\lambda} \partial_\rho \left(\partial_\beta \bar{N}^{\mu\nu]\rho|\beta}_k - \partial^\mu \bar{N}^{\nu]\rho\sigma|\tau}_k \sigma_{\sigma\tau}\right) - \\ &\quad - \sigma^{\alpha[\lambda} \partial^\rho \partial_\rho \bar{N}^{\mu\nu]\sigma|\tau}_k \sigma_{\sigma\tau}.\end{aligned} \tag{B.23}$$

where all the bar quantities are invariant. Since α_k is invariant, it depends on $t_{\mu\nu\lambda|\alpha}$ only through the curvature and its derivatives, such that

$$\frac{\delta^R \alpha_k}{\delta t_{\mu\nu\lambda|\alpha}} = \partial_\xi \partial_\beta \Delta_k^{\mu\nu\lambda\xi|\alpha\beta} , \tag{B.24}$$

where $\Delta_k^{\mu\nu\lambda\xi|\alpha\beta}$ has the mixed symmetry of the curvature tensor. The part from (B.23) involving $\bar{N}$ has a form similar to that of the right-hand side of (B.24). Then, $\delta \bar{L}_{k+1}^{\mu\nu\lambda|\alpha}$ must be expressed in the same manner, i.e.,

$$\delta \bar{L}_{k+1}^{\mu\nu\lambda|\alpha} = \partial_\xi \partial_\beta \Omega_k^{\mu\nu\lambda\xi|\alpha\beta} , \tag{B.25}$$

for a Ω with the mixed symmetry of the curvature. The equation (B.25) shows that for a given α the object $\bar{L}_{k+1}^{\mu\nu\lambda|\alpha}$ belongs to $H_{k+1}^{D-3}(\delta|d)$. As $H_{k+1}^{D-3}(\delta|d) \simeq H_{k+2}^{D-2}(\delta|d) \simeq H_{k+3}^{D-1}(\delta|d) \simeq H_{k+4}^{D}(\delta|d)$ (see [28], theorem 8.1) and $H_{k+4}^{D}(\delta|d) \simeq 0$, the equation (B.25) implies that

$$\bar{L}_{k+1}^{\mu\nu\lambda|\alpha} = \delta R_{k+2}^{\mu\nu\lambda\alpha} + \partial_\rho U_{k+1}^{\rho\mu\nu\lambda\alpha} , \tag{B.26}$$

where $U_{k+1}^{\rho\mu\nu\lambda\alpha}$ is antisymmetric in ρ, μ, ν and λ.

Now, we prove the theorem in the case $k \leq D$ by induction. This is, we assume that the theorem is valid in antighost number $(k+4)$ and in form degree D, and show that it holds in antighost number k and in form degree D. In agreement with the induction hypothesis, $R_{k+2}^{\mu\nu\lambda\alpha}$ and $U_{k+1}^{\rho\mu\nu\lambda\alpha}$ from (B.26) can be assumed to be invariant. On the other hand, $\bar{L}_{k+1}^{\mu\nu\lambda|\alpha}$ must verify the mixed symmetry of the tensor field $t^{\mu\nu\lambda|\alpha}$, i.e., $\bar{L}_{k+1}^{[\mu\nu\lambda|\alpha]} = 0$, which further implies that

$$\delta R_{k+2}^{[\mu\nu\lambda\alpha]} + \partial_\rho U_{k+1}^{\rho[\mu\nu\lambda\alpha]} = 0 . \tag{B.27}$$

Acting with δ on (B.27), we obtain $\partial_\rho \left(\delta U_{k+1}^{\rho[\mu\nu\lambda\alpha]}\right) = 0$, such that

$$\delta U_{k+1}^{\rho[\mu\nu\lambda\alpha]} = \partial_\tau V_k^{\rho\tau\mu\nu\lambda\alpha} , \tag{B.28}$$

where $V_k^{\rho\tau\mu\nu\lambda\alpha}$ is separately antisymmetric in $\rho\tau$ and $\mu\nu\lambda\alpha$. The equation (B.28) shows that for fixed ρ and α, $U_{k+1}^{\rho[\mu\nu\lambda\alpha]}$ pertains to $H_{k+1}^{D-3}(\delta|d)$, that is finally found isomorphic to $H_{k+4}^{D}(\delta|d) \simeq 0$, so $U_{k+1}^{\rho[\mu\nu\lambda\alpha]}$ is trivial

$$U_{k+1}^{\rho[\mu\nu\lambda\alpha]} = \delta W_{k+2}^{\rho\mu\nu\lambda\alpha} + \partial_\sigma S_{k+1}^{\mu\nu\lambda\alpha|\sigma\rho} , \tag{B.29}$$

with $S_{k+1}^{\mu\nu\lambda\alpha|\sigma\rho}$ having the mixed symmetry of the curvature tensor. Using again the induction hypothesis, we can assume that $W_{k+2}^{\rho\mu\nu\lambda\alpha}$ and $S_{k+1}^{\mu\nu\lambda\alpha|\sigma\rho}$ are invariant. In order to reconstruct α_k through the homotopy formula (B.14), we need to compute $\delta \bar{L}_{k+1}^{\mu\nu\lambda|\alpha}$ by means of formula (B.26), so eventually we need to calculate $\partial_\rho U_{k+1}^{\rho\mu\nu\lambda\alpha}$. In this respect we will use the equation (B.29) and the identity (that holds only for some tensor that is antisymmetric in its first four indices)

$$U_{k+1}^{\rho\mu\nu\lambda\alpha} = \frac{1}{4}\left(U_{k+1}^{\rho[\mu\nu\lambda\alpha]} + U_{k+1}^{\nu[\alpha\rho\lambda\mu]} + 3U_{k+1}^{\alpha[\nu\rho\lambda\mu]} + U_{k+1}^{\lambda[\mu\nu\alpha\rho]} + U_{k+1}^{\mu[\nu\alpha\rho\lambda]}\right) , \tag{B.30}$$

and obtain that

$$\partial_\rho U_{k+1}^{\rho\mu\nu\lambda\alpha} = \delta \tilde{W}_{k+2}^{\mu\nu\lambda\alpha} + \partial_\sigma \partial_\rho \left(S_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha} + E_{k+1}^{\lambda\mu\nu\rho\sigma\alpha} \right), \tag{B.31}$$

where

$$\begin{aligned} E_{k+1}^{\lambda\mu\nu\rho\sigma\alpha} &= \frac{1}{4}\Big(S_{k+1}^{\alpha\rho\lambda\mu|\sigma\nu} + S_{k+1}^{\nu\alpha\rho\lambda|\sigma\mu} + S_{k+1}^{\mu\nu\alpha\rho|\sigma\lambda} + S_{k+1}^{\lambda\mu\nu\alpha|\sigma\rho} - \\ &\quad - S_{k+1}^{\sigma\rho\lambda\mu|\alpha\nu} + S_{k+1}^{\nu\sigma\rho\lambda|\alpha\mu} + S_{k+1}^{\mu\nu\sigma\rho|\alpha\lambda} + S_{k+1}^{\lambda\mu\nu\sigma|\alpha\rho} \Big). \end{aligned} \tag{B.32}$$

Obviously, $E_{k+1}^{\lambda\mu\nu\rho\sigma\alpha}$ is invariant as $S_{k+1}^{\alpha\rho\lambda\mu|\sigma\nu}$ is invariant. By direct handling it can be shown that $E_{k+1}^{\lambda\mu\nu\rho\sigma\alpha} = E_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha}$ in the sense that indeed it has the mixed symmetry of the curvature tensor. Denoting by $\tilde{S}_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha}$ the term $S_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha} + E_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha}$, it is simple to see that it is invariant and possesses the mixed symmetry of the curvature. Using the above notation and inserting (B.31) in (B.26), it results that

$$\bar{L}_{k+1}^{\mu\nu\lambda|\alpha} = \delta \tilde{R}_{k+2}^{\mu\nu\lambda\alpha} + \partial_\sigma \partial_\rho \tilde{S}_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha}. \tag{B.33}$$

With the help of (B.20)–(B.23) and (B.33), the formula (B.14) becomes

$$\begin{aligned} \alpha_k = \delta \Big[\int_0^1 d\tau \Big(& \bar{G}_{k-3}^{\alpha} C_\alpha^* + \bar{K}_{k-2}^{\alpha\beta} \mathcal{C}'^*_{\alpha\beta} + \bar{M}_{k-1}^{\mu\nu|\alpha} \mathcal{G}'^*_{\mu\nu|\alpha} + \\ & + \bar{N}_k^{\mu\nu\lambda|\alpha} t^*_{\mu\nu\lambda|\alpha} + \left(\partial_\sigma \partial_\rho \tilde{S}_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha} \right) t_{\mu\nu\lambda|\alpha} \Big) \Big] + \partial_\mu \sigma_k^\mu . \end{aligned} \tag{B.34}$$

The last term in the argument of δ can be written in the form

$$\left(\partial_\sigma \partial_\rho \tilde{S}_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha} \right) t_{\mu\nu\lambda|\alpha} = -\frac{1}{8} \tilde{S}_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha} F_{\lambda\mu\nu\rho|\sigma\alpha} + \partial_\mu \phi^\mu , \tag{B.35}$$

so finally we arrive at

$$\begin{aligned} \alpha_k = \delta \Big[\int_0^1 d\tau \Big(& \bar{G}_{k-3}^{\alpha} C_\alpha^* + \bar{K}_{k-2}^{\alpha\beta} \mathcal{C}'^*_{\alpha\beta} + \bar{M}_{k-1}^{\mu\nu|\alpha} \mathcal{G}'^*_{\mu\nu|\alpha} + \\ & + \bar{N}_k^{\mu\nu\lambda|\alpha} t^*_{\mu\nu\lambda|\alpha} - \frac{1}{8} \tilde{S}_{k+1}^{\lambda\mu\nu\rho|\sigma\alpha} F_{\lambda\mu\nu\rho|\sigma\alpha} \Big) \Big] + \partial_\mu \psi_k^\mu . \end{aligned} \tag{B.36}$$

We observe that all the terms from the integrand are invariant. In order to prove that the current ψ_k^μ can also be taken invariant, we switch (B.36) to the original form notation

$$\alpha_k^D = \delta \lambda_{k+1}^D + d\lambda_k^{D-1}, \tag{B.37}$$

(where λ_k^{D-1} is dual to ψ_k^μ). As α_k^D is by assumption invariant and we have shown that λ_{k+1}^D can be taken invariant, (B.37) becomes

$$\beta_k^D = d\lambda_k^{D-1}. \tag{B.38}$$

It states that the invariant polynomial $\beta_k^D = \alpha_k^D - \delta\lambda_{k+1}^D$, of form degree D and of strictly positive antighost number, is d-exact. Then, in agreement with the theorem 1 in form degree D (see the paragraph following this theorem), we can take λ_k^{D-1} (or, which is the same, ψ_k^{μ}) to be invariant.

In conclusion, the induction hypothesis for antighost number $(k+4)$ leads to the same property for antighost number k, which proves the theorem for all $k \geq 4$ since we have shown that it holds for $k > D$. $\square$

The most important consequence of the last theorem is the validity of the result (5.31) on the triviality of $H^{\text{inv}}(\delta|d)$ in antighost number strictly greater than four.

C. Elimination of $(a_k)_{k>4}$ from the first-order deformation

In order to show that we can eliminate all a_k with $k > 4$ from the first order deformation (5.2) only by trivial redefinitions (or, in other words, that formula (5.34) is correct), we start from the equations (5.3)–(5.5) written in dual form notations

$$\begin{aligned} \gamma a_I &= 0\,, && \text{(C.1)}\\ \delta a_I + \gamma a_{I-1} &= dm_{I-1}\,, && \text{(C.2)}\\ &\vdots \\ \delta a_k + \gamma a_{k-1} &= dm_{k-1}\,, && \text{(C.3)}\\ &\vdots \\ \delta a_1 + \gamma a_0 &= dm_0\,, && \text{(C.4)} \end{aligned}$$

where $\text{agh}(a_k) = k = \text{pgh}(a_k)$ for $0 < k < I$ and $\text{agh}(m_k) = k$, $\text{pgh}(m_k) = k+1$ for $0 < k < I-1$. By hypothesis, we take $I > 4$. All a_k are D-forms and all m_k are $(D-1)$-forms. We have seen that the solution to the top equation is

$$a_I = \sum \alpha_J \omega^J\,, \tag{C.5}$$

where all α_J are invariant polynomials with $\text{agh}(\alpha_J) = I$ and the form coefficients of all ω^J are elements of pure ghost number I of a basis in the space of polynomials in $\mathcal{F}_{\lambda\mu\nu\alpha}$ and C_ν. Applying γ on (C.2) and using (C.1) we find that $-d(\gamma m_{I-1}) = 0$, such that the triviality of the cohomology of d implies that

$$\gamma m_{I-1} + dn_{I-1} = 0\,, \tag{C.6}$$

where $\text{agh}(n_{I-1}) = I-1 > 0$, $\text{pgh}(n_{I-1}) = I+1$, $\deg(n_{I-1}) = D-2$. From the corollary 1, it follows that we can replace (C.6) with the equation

$$\gamma m_{I-1} = 0\,, \tag{C.7}$$

and hence

$$m_{I-1} = \sum \beta_J \omega^J\,, \tag{C.8}$$

where all β_J are invariant polynomials with $\text{agh}(\beta_J) = I-1$ and $\text{pgh}\left(\omega^J\right) = I$ for all J. Replacing (C.5) and (C.8) in (C.2), we find that

$$\sum \pm (\delta\alpha_J)\,\omega^J + \sum \mp (d\beta_J)\,\omega^J = \sum \beta_J d\omega^J - \gamma a_{I-1}\,. \tag{C.9}$$

According to the relations 5.18 and 5.24, we have that $d\omega^J = \gamma\hat{\omega}^J$ (so $\beta_J d\omega^J = \gamma\left(\beta_J\hat{\omega}^J\right)$), such that (C.9) becomes

$$\sum\left(\pm\delta\alpha_J \mp d\beta_J\right)\omega^J = \gamma\left(\sum\beta_J\hat{\omega}^J - a_{I-1}\right). \tag{C.10}$$

But both $\delta\alpha_J$ and $d\beta_J$ are invariant polynomials of antighost number $(I-1) > 0$, and hence $\beta'_J = (\pm\delta\alpha_J \mp d\beta_J)$ is an invariant polynomial with the same antighost number. From (C.10) we obtain that the invariant polynomials β'_J appearing in its left-hand side must vanish for all J

$$\beta'_J = \pm\delta\alpha_J \mp d\beta_J = 0\,, \qquad \text{for all } J\,, \tag{C.11}$$

since a term of the type $\beta'_J\omega^J$ (with β'_J invariant) is γ-exact if and only if it vanishes. Since by assumption $\mathrm{agh}\left(\alpha_J\right) = I > 4$ and $H_I\left(\delta|d\right) = 0$ for $I > 4$, from (C.11) we find that

$$\alpha_J = \delta\rho_J + d\nu_J\,, \qquad \text{for all } J\,, \tag{C.12}$$

where $\mathrm{agh}\left(\rho_J\right) = I+1$ and $\mathrm{agh}\left(\nu_J\right) = I$. Relying now on the theorem 3, we get that we can take all ρ_J and ν_J to be invariant polynomials. Inserting now (C.12) in (C.5), after some direct computation we find that

$$\begin{aligned} a_I &= \sum\left(\delta\rho_J\right)\omega^J + \sum\left(d\nu_J\right)\omega^J \\ &= s\left(\sum\pm\rho_J\omega^J\right) + d\left(\sum\pm\nu_J\omega^J\right) + \gamma\left(\sum\mp\nu_J\hat{\omega}^J\right). \end{aligned} \tag{C.13}$$

Since a_I is unique up to trivial, γ-exact contributions, the last formula shows that a_I can be redefined by adding to it only trivial (and invariant) quantities such that

$$a_I = 0\,. \tag{C.14}$$

Inserting (C.14) in (C.2), the last equation becomes

$$\gamma a_{I-1} = dm_{I-1}\,. \tag{C.15}$$

where $\mathrm{agh}\left(a_{I-1}\right) = I-1 > 0$, $\mathrm{gh}\left(a_{I-1}\right) = 0$ and $\deg\left(a_{I-1}\right) = D$, so by corollary 1 it can be substituted by trivial redefinitions with the equation

$$\gamma a_{I-1} = 0\,. \tag{C.16}$$

Reprising now exactly the same procedure like before, but in relation with the equation (C.16) and

$$\delta a_{I-1} + \gamma a_{I-2} = db_{I-2}\,, \tag{C.17}$$

we find that we can take $a_{I-1} = 0$ by trivial redefinitions only. The elimination procedure stops at $k = 4$ in the tower (C.1)–(C.4) since for $k = 4$ we cannot pass from (C.11) to (C.12) as $H_4\left(\delta|d\right) \neq 0$. In conclusion, we can replace a_k with $a_k = 0$ by trivial redefinitions for all $k \geq 5$ in the tower (C.1)–(C.4), such that the first-order deformation can always be taken to end at $I = 4$ (formula (5.34)), with a_4 from $H\left(\gamma\right)$, $\gamma a_4 = 0$.

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